Winkler subsoil model for foundation pad

IDEA StatiCa

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1 Winkler subsoil model

Winkler subsoil model provides stiffness in compression corresponding to the stiffness of concrete foundation pad in local bearing under each node of a base plate. The springs are effective only in compression and do not restrain possible separation of a base plate from concrete foundation pad. Using this simplification, the number of nodes and therefore the calculation time are drastically decreased. However, concrete failure modes of anchors and shear lug and concrete bearing strength must be determined analytically according to relevant codes.

2 Concrete bearing stiffness

2.1 EC model

The stiffness of T-stub in compression including grout is specified in EN 1993-1-8, Table 6.11 as:

$$k_{13} = \frac{E_c \sqrt{b_{eff} \cdot l_{eff}}}{1.275E}$$

where:

- E_c – Young modulus of elasticity of concrete [Pa]
- b_{eff} – width of the T-stub [m]
- l_{eff} – length of the T-stub [m]
- E – Young modulus of elasticity of steel [Pa]

The resulting unit is [m].

For detailed information see Martin Steenhuis, František Wald, Zdeněk Sokol, Jan Stark: *Concrete in compression and base plate in bending*; available at: `http://heronjournal.nl/53-12/3.pdf`.

2.2 Soil mechanics

Winkler subsoil model is often used in soil mechanics for infinite subsoil under a foundation. The stiffness is specified as:

$$k = \frac{\Delta p}{\Delta s}$$

where:

- Δp – pressure increment [N/m^2]
- Δs – deformation increment [m]

The stiffness k can also be calculated as:

$$k = E \cdot \frac{2}{d \cdot \ln\left(1 + 2 \cdot \frac{h}{d}\right)} \cdot \sqrt{\frac{A_{ref}}{A_{eff}}}$$

where:

- E – modulus of elasticity [Pa]
- d – width of the loaded area [m]
- h – height of the layer [m]
- A_{ref} – reference area [m^2]
- A_{eff} – loaded area [m^2]

The resulting unit is [N/m^3]. For solid rock, the stiffness of $k = 5000 \div 15000$ kN/mm^3 is recommended.

2.3 Numerical experiments

The stiffness of concrete in bearing is nearly impossible to measure experimentally because it is very high. Measurement of both stress in concrete and deformation is difficult.

Therefore, numerical experiments were performed to determine the stiffness of concrete in bearing. Two software were used – Midas FEA and ATENA. Some of the used models are shown in Figures 1 and 2. Various parameters were selected:

- Concrete grade
- Concrete pad dimensions
- Column cross-section shape and dimensions
- Base plate dimensions
- Subsoil stiffness under the foundation pad
- Compressive load magnitude

The parametric study led to graphs of stiffness in dependence on the selected variable. It was found that important variables are concrete grade, concrete pad height, and base plate dimensions. On the other hand, subsoil stiffness under the foundation pad or compressive load magnitude before the local bearing resistance is reached are irrelevant.

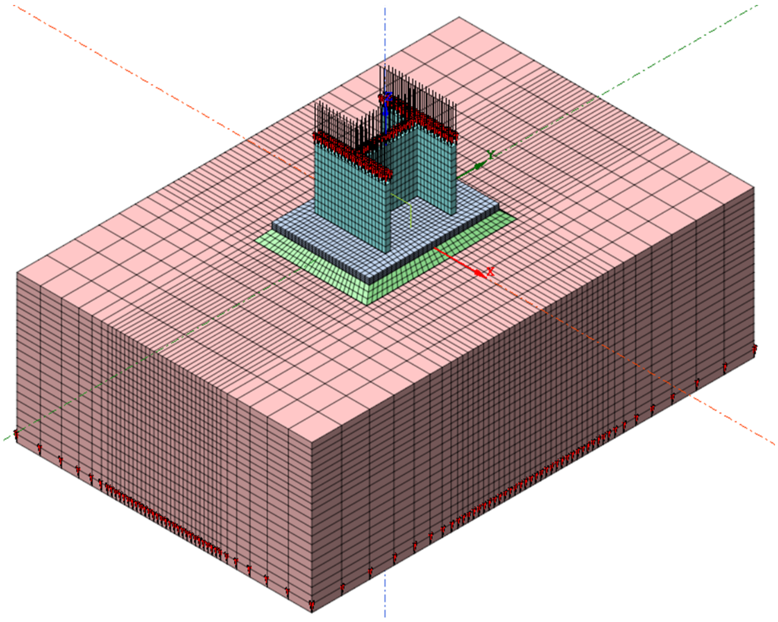

Figure 1: Typical finite element model to determine concrete stiffness in bearing

2.4 Used analytical stiffness

The formula for concrete in bearing stiffness is highly inspired by stiffness by EN 1993-1-8 and findings from soil mechanics. Afterwards, curve-fitting was used especially to take into account the findings from numerical experiments.

$$k = \frac{E_c}{(\alpha_1 + \nu)\sqrt{\frac{A_{eff}}{A_{ref}}}} \left(\frac{1}{\frac{h}{\alpha_2 d} + \alpha_3} + \alpha_4 \right)$$

where:

- E_c – Young modulus of elasticity of concrete [Pa]
- ν – Poisson coefficient of concrete [-]
- A_{eff} – base plate area in contact with concrete [m^2]
- $A_{ref} = 1$ m^2 – reference area
- h – concrete pad height [m]
- d – width of the effective area [m]
- parameters for curve-fitting:

  $\alpha_1 = 1.65$

  $\alpha_2 = 0.5$

  $\alpha_3 = 0.3$

  $\alpha_3 = 1.0$

3 Comparison

The shortcoming of the Winkler subsoil model is that the stiffness under the base plate is not constant. While the highest deformation is usually in the middle, the principal stress is the largest in the corner; see Figure 2.

The effect of base plate area is shown in Figure 3. The local bearing stiffness of concrete is decreasing with increasing base plate area.

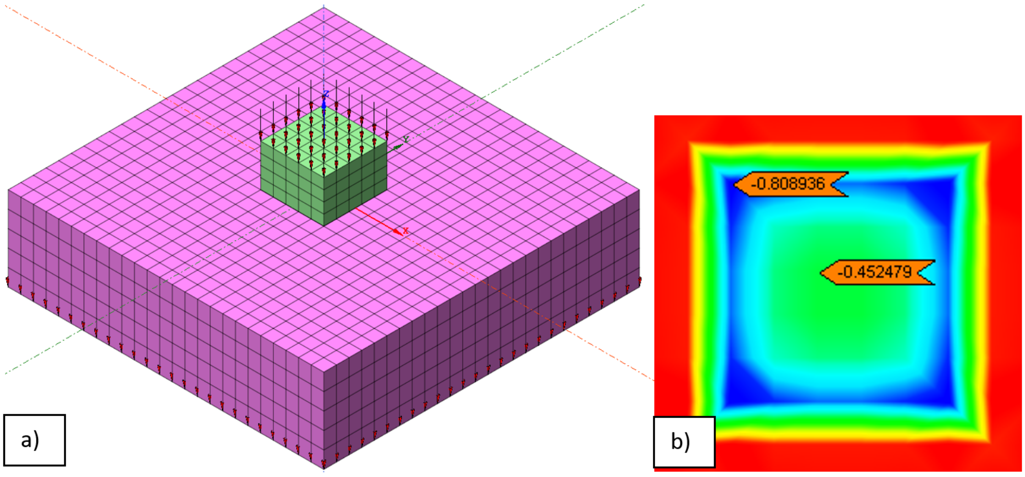

Figure 2: Simulation of rigid base plate: a) Model of steel block being pushed into concrete pad, b) principal stress in concrete

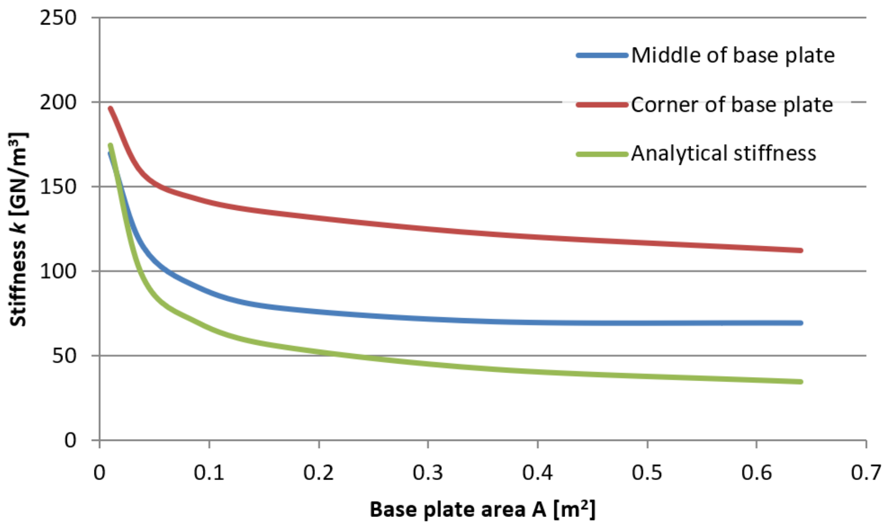

Figure 3: Comparison of stiffness by analytical formula with numerical models for rigid base plate

The effect of concrete foundation pad height is shown in Figure 4. The stiffness is decreasing with increasing height of concrete foundation pad.

However, as can be seen from the comparison in Figure 5, the stresses under the base plate using solid 3D elements and Winkler subsoil are nearly identical. Also, it was found that the concrete local bearing stiffness does not significantly affect the results. Even the change by hundreds of percents does not change concrete stress or anchor forces significantly. The concrete stiffness is still very high compared to stiffness of anchors.

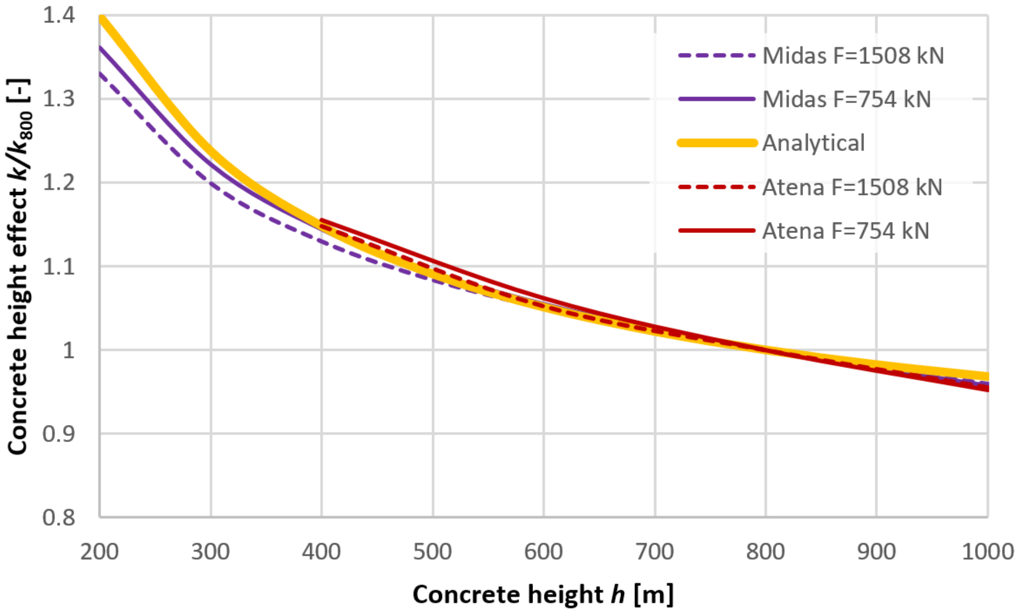

Figure 4: Concrete local bearing stiffness normalized to the stiffness of concrete pad with the height of 800 mm; comparison of numerical models and analytical formula

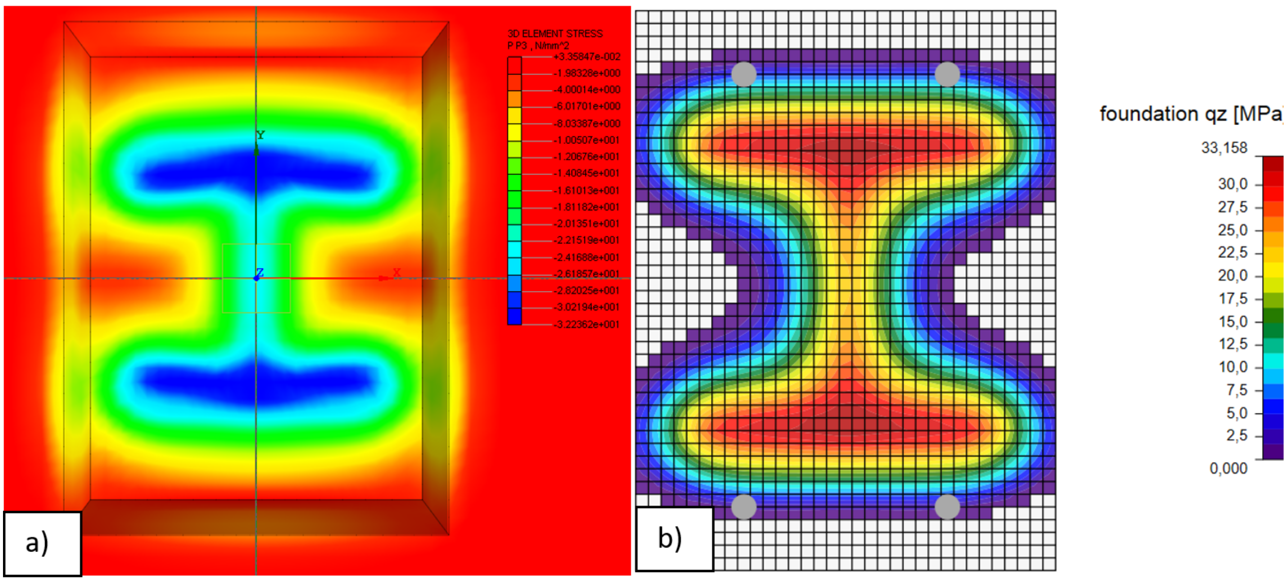

Figure 5: a) principal stress in the concrete 3D model, b) compressive stress under base plate using Winkler subsoil model

4 Implementation in CBFEM method

The stiffness of Winkler subsoil model is estimated before the finite element analysis is performed. The concrete stiffness is impossible to determine exactly before the calculation starts due to unknown local bearing area A_{eff} and concrete supporting area A_{c1} and subsequently the concentration factor k_j. The local bearing area is estimated as:

$$A_{eff} = 4 \cdot b_w \cdot c + (b_h - c) \cdot 2 \cdot c$$

where:

- b_w – base plate width
- b_h – base plate height
- $c = t_p \cdot \sqrt{\frac{f_y}{3 \cdot f_{jd} \cdot \gamma_{M0}}}$ – additional bearing width taken from EN 1993-1-8
- $f_{jd} = \beta_j \cdot k_j \cdot \frac{f_{ck}}{\gamma_c}$ – design concrete bearing strength taken from EN 1993-1-8 and EN 1992-1-1
- $\beta_j = 0.6$ – foundation joint material coefficient taken from EN 1993-1-8
- f_{ck} – characteristic concrete strength
- $k_j = \sqrt{\frac{A_{c1}}{A_{eff}}} \leq 3$ – concentration factor taken from EN 1992-1-1 estimated as $k = 2$
- A_{c1} – concrete supporting area
- $\gamma_c = 1.5$ – material safety factor of concrete
- $\gamma_{M0} = 1.0$ – material safety factor of steel

The width of the effective area, d, is taken as the width of the base plate (shorter dimension of rectangular base plate).