

Serviceability Limit State Evaluation in Discontinuity Regions

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Abstract. Discontinuity Region Design method was recently extended to allow assessment of serviceability limit states (SLS) for regions of concrete structural members where the Bernoulli-Navier hypothesis does not hold, such as dapped ends, openings, frame corners, etc. The method uses material models which consider the impact of short- and long-term loading effects (creep) as well as the influence of tension stiffening, which are calculated from reinforcement ratios. The method can be used to perform assessment of stress limitation SLS as well as to calculate crack widths. Crack width calculations for both stabilized and non-stabilized cracks have been compared with real-world experiments. Calculations regarding deflection and strain in concrete and concrete rebars are compared with analytical calculations.

Introduction

Regions of discontinuity are characterized by the fact that the Bernoulli-Navier hypothesis, which assumes retention of cross-section planarity, does not hold due to abrupt changes in geometry or a high concentration of applied load. Such concrete details (e.g. openings, dapped ends, beam ends with supports, etc.) are very often the most critical parts of the concrete member and at the same time, it is not possible to conduct evaluations in terms of “sections” or “points” as described in the relevant standard [1].

Historically, such discontinuity regions had to be assessed by semi-empirical design rules and later by strut-and-tie models [2, 3, 4] and stress-field models [5, 6, 7, 8, 9] that are widely used in modern design codes, by designers and some advanced computational tools today. The “truss analogy” (strut-and-tie) method requires a model to be set up with a topology composed of concrete struts and reinforcement ties. Despite its disadvantage, the method is very fast, recommended by standards, and can be used to evaluate ultimate limit states (ULS) in a generally reliable manner. Of course, what the model is not capable of evaluating is serviceability limit states (SLS).

The compression field model, as presented in [9] and [10], can be considered a generalized truss analogy method in which, however, real regions loaded by stress are considered instead of the resultant force from a strut-and-tie model. Discontinuity Region Design (DRD) method was developed to overcome the limitations of the classic design tools and computational models while keeping the advantages of the strut-and-tie and stress-field models. The method was implemented into a user-friendly commercial software IDEA StatiCa Detail [11, 12] that was first presented at the end of 2017 and its first version included ULS evaluation only. It has however recently been updated to include SLS assessment capability. The implementation of the compression field model itself is not described in this article (it is described in [9, 10, 13]), which focuses exclusively on SLS evaluation.

Basic Principles

As mentioned above, the compression field model implemented within the finite element method is used for evaluation. The only assumption is the action of compression in concrete and strain in reinforcement.

The SLS calculation itself is conducted using two computational models:

The short-term model

This model describes the immediate response of a structure to loading. For both components of load, i.e. for both permanent and variable load, a linear stress-strain curve is considered, with the secant modulus of elasticity E_{cm} .

The long-term (compound) model

This model describes the long-term response of a structure to loading. With regard to the fact that during the calculations both permanent (long-term) and variable (short-term) loads are considered, the modulus of elasticity changes in relation to the applied load. For the long-term loading of concrete the effective elastic modulus $E_{c,eff}$ is applied, while for short-term loading the secant modulus of elasticity E_{cm} is used.

Concrete. The effective elastic modulus $E_{c,eff}$ considers the effect of creep and shrinkage with the aid of the basic creep coefficient φ . $E_{c,eff} = E_{cm} / (1 + \varphi)$

Reinforcement. In the default settings, an idealized bilinear stress-strain curve is applied to reinforcement “without tension stiffening” defined in the relevant standard (Fig 2.). This curve is merely defined by basic characteristics of the reinforcement which are already known during the design phase (such as strength class and ductility). The effect of mutually acting stiffness of concrete – tension stiffening – is then taken into account by modifying the stress-strain curve. The resultant model considers the mean stiffness of mutually acting reinforcement and concrete.

The implementation of tension stiffening differentiates between cases when stabilized and non-stabilized cracks occurs.

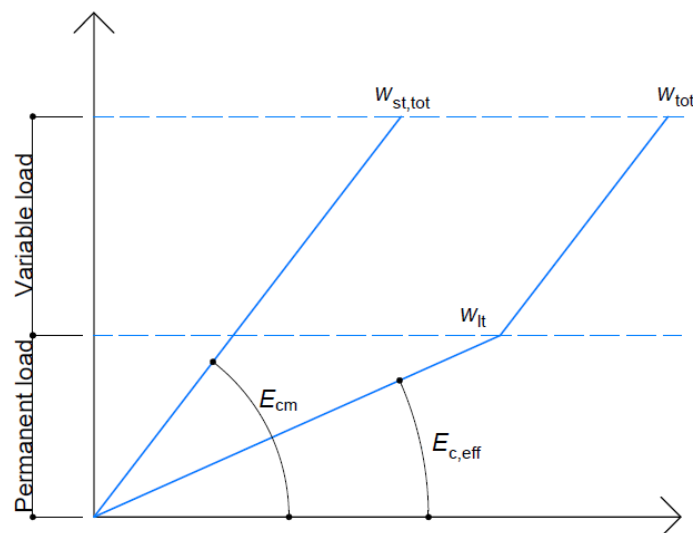


Fig. 1: Elastic moduli of concrete for short-term and long-term loads

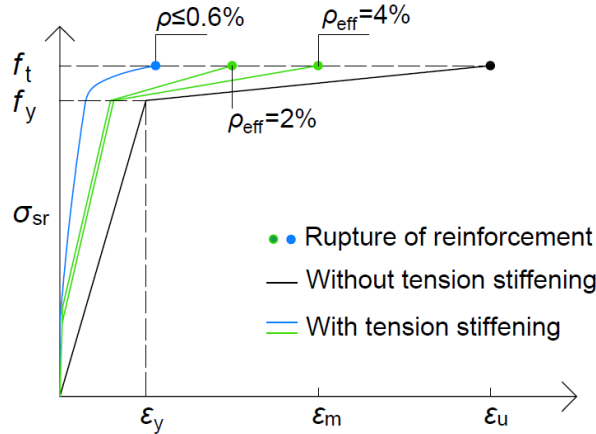


Fig. 2: Stress-strain diagram for reinforcement with and without tension stiffening [9]

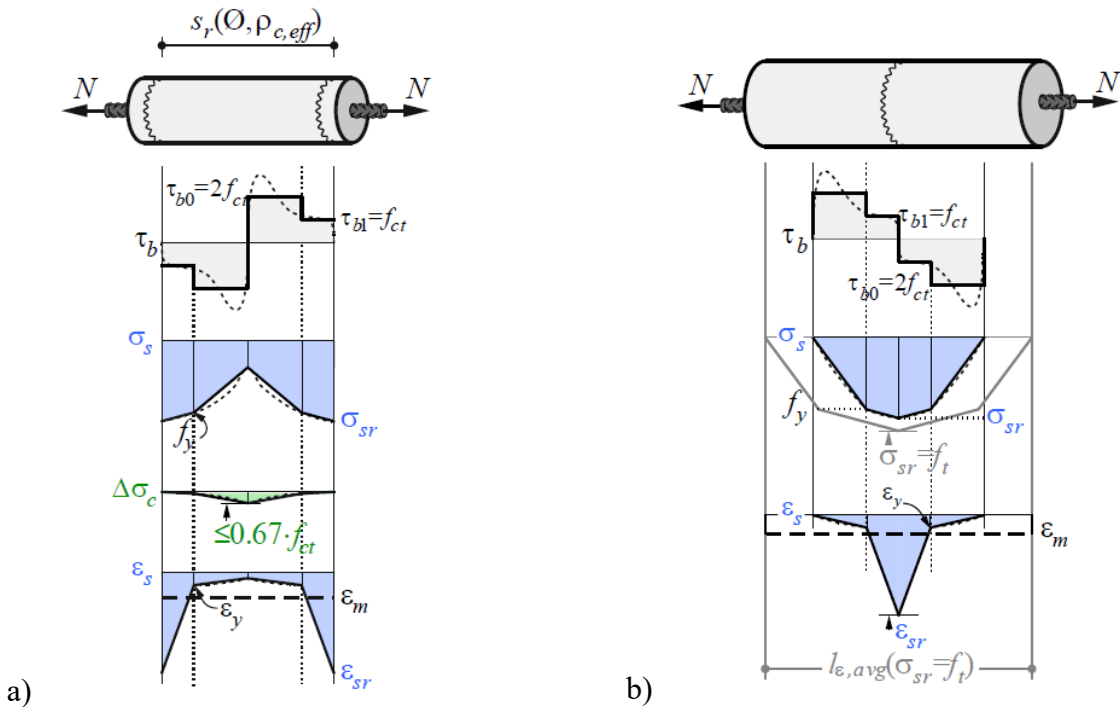


Fig. 3: a) A “TCM” element for stabilized crack growth, b) “Pull-Out” assumption for non-stabilized crack growth [9]

Stabilized crack growth. In the case of fully developed stabilized cracks the Tension Chord Model (TCM) is used to calculate tension stiffening [14] – Fig. 3a. With regard to the fact that in the TCM tension stiffening depends on reinforcement area and its assignment to each reinforcement bar or layer, the determination of the pertinent (mutually acting) concrete surface under effective strain is critical. For this reason, automatic spatial identification of the corresponding effective concrete surface mutually acting in tension for an arbitrary reinforcement configuration was implemented. The maximum distance between cracks s_{r0} stabilizes at a value at which the stress in concrete between two neighboring cracks does not reach the stress value of the crack initiation limit state. In this way the growth of further cracks is ended. The default settings of IDEA StatiCa Detail assume an average crack distance of $0.67 s_{r0}$.

Non-stabilized crack growth. Non-stabilized crack growth is considered for local cracks triggered by geometrical discontinuities (e.g. regions where the cross-section changes, concave corners, etc.) as well as regions with a reinforcement ratio. In case that the reinforcement ratio is lower than ρ_{cr} ($\sim 0.6\%$), the reinforcement is not capable of transferring load at the limit state of cracking without exceeding yield strength.

Cracks in regions with reinforcement ratios lower than ρ_{cr} are either generated by non-mechanical phenomena (e.g. shrinkage) or by crack progression which is directed by other reinforcement components. In such cases, the crack is non-stabilized and tension stiffening is considered with the aid of the “Pull-Out” model – Fig. 3b. This model analyses the behavior of individual cracks without considering the mechanical interaction between other cracks, neglects the behavior of concrete in tension and assumes the same ideally rigid-plastic behavior in cohesion used in the Tension Chord Model.

Given the fact that the crack spacing isn't known for a non-fully developed crack pattern, the average strain is computed for any load level over the distance between points with zero slip when the reinforcing bar reaches its tensile strength at the crack.

Table 1: Comparison of maximum stress calculations

	$\sigma_{c,st}$ [MPa]	$\sigma_{s,st}$ [MPa]	$\sigma_{c,lt}$ [MPa]	$\sigma_{s,lt}$ [MPa]
IDEA StatiCa RCS	-7.1	290.4	-4.2	299.5
IDEA StatiCa Detail	-4.9	286.8	-4.7	287.6
IDEA StatiCa Detail – all permanent	-4.9	286.8	-3.4	296.4

Relation to Standards

The way the finite element model is assembled is standard-independent, but evaluations and the interpretation of results are all carried out according to currently valid standards [1]. IDEA StatiCa Detail enables the evaluation of stress limitation (Section 7.2), crack initiation and width (Section 7.3) and deformation (Section 7.4) in compliance with the Eurocode 2 standard [1].

Stress limitation. Stress limitation is implemented according to Section 7.2 of the standard [1] – on the basis of the assumptions described in paragraph 2 of that section. The physical model calculates the tension on the finite element mesh and compares it with the limiting stress of concrete described in 7.2 (1)(2)[1]. In the evaluation of stress limitation the stress is simultaneously calculated along the length of the rebar insert and compared with the limiting stress of the reinforcement 7.2 (3)[1].

The stress in concrete σ_c and in reinforcement σ_s is checked against the limit stress from the standard.

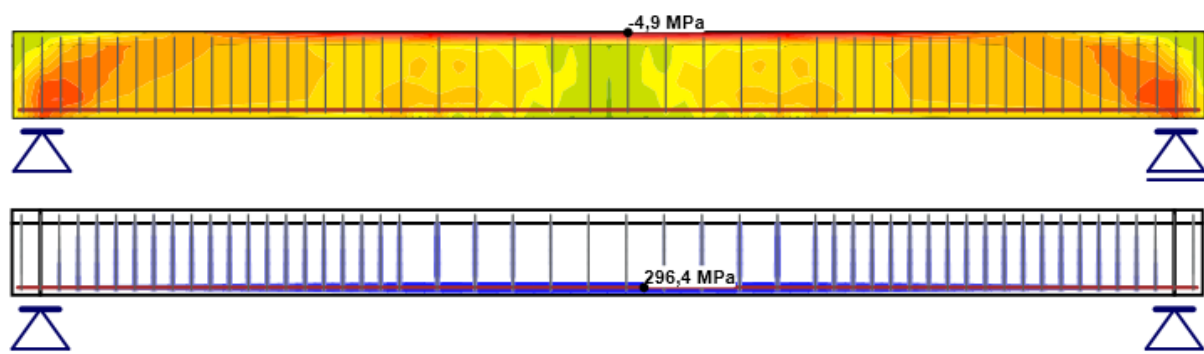


Fig. 4: Resultant maximum stress in concrete and in reinforcement

Evaluation of crack initiation and width. The calculation of crack width is divided into the calculation of stabilized (uniformly distributed – e.g. along the lower edge of a beam) and non-stabilized (isolated) cracks which initiate at sites of high stress concentration (e.g. at locations where there are sharp edges, corners of dapped ends, shear cracks in walls). All stress and strain calculations are performed on the basis of assumptions stated in Section 2. The calculation of cracks is itself conducted based on the method described in [9]

The calculation of crack width is performed for permanent load. Two main models are available, as described in parts 2.3 and 2.4:

- Model of stabilized crack growth
- Model of non-stabilized crack growth

Both these models depend on the type of reinforcement, on the automatically calculated reinforcement ratio and subsequently on the tension stiffening of every individual 1D element used to model the reinforcement. The width of a crack perpendicular to the orientation of the reinforcement w_b is calculated on the basis of the aforementioned models via tension stiffening using the integration of strain over reinforcement. For regions with stabilized crack growth, mean values of strain of the reinforcement are calculated and integrated over the mean crack distance – Fig. 5a. In the case of non-stabilized crack growth, the width w_b is calculated according to the procedure depicted in Fig. 5b on the basis of the maximum stress in the reinforcement, which in this case is more reliable than mean strain.

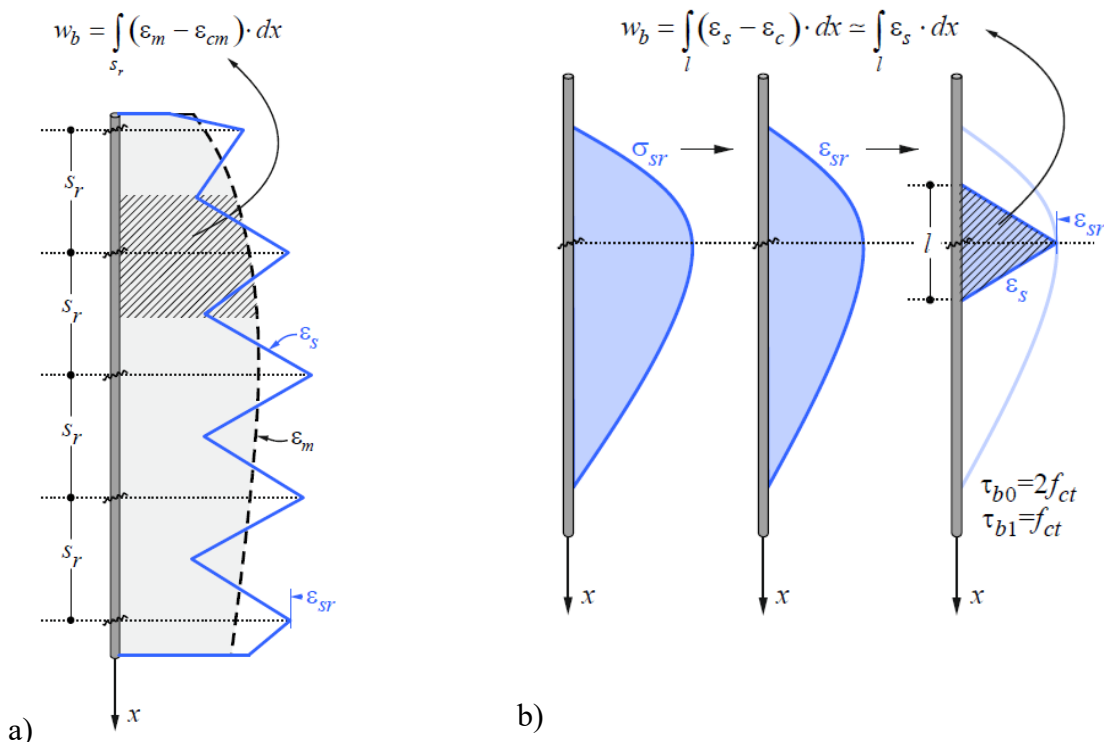


Fig. 5: a) Crack width perpendicular to reinforcement for stabilized crack growth.
 b) Crack width perpendicular to reinforcement for non-stabilized crack growth [9]

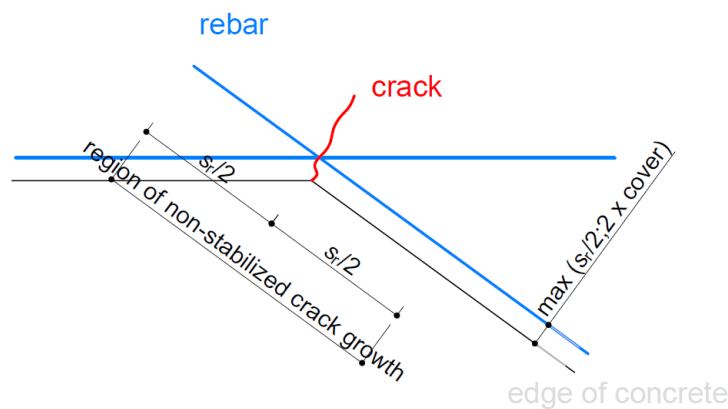


Fig.6: Special situations occur at concave corners of details or walls

If a crack appears in a given concave corner, its position is then determined by that corner. The direction of the crack is given by the direction of the main strain at that corner, and the crack width calculation is considered to be non-stabilized, even if the reinforcement area of the adjacent rebar assumes stabilized crack growth.

The orientation of cracks is then determined by the direction of main strain from the same region and the width of cracks w is adjusted with regard to the difference in angle between the orientation of the reinforcement and the direction of the main strain in the fiber of the concrete adjacent to the respective reinforcement bar.

This approach does not correspond to the true position of cracks, but still produces relevant results which can be compared with values required by the code. Basically, the results can be interpreted as an opening of a crack with the assumption that it initiates at the respective location.

The above method does not enable the evaluation of the initiation and widths of cracks in areas of concrete where reinforcement is completely absent.

Evaluation of deformation. In terms of evaluation of deformation, calculations are performed to determine short-term deflections from the total load, deflections from long-term load under the influence of creep, increases in deflection from short-term (variable) loading, and total deflection. The evaluation is carried out with respect to a limit value set by the user. Deflection can be evaluated for walls or statically determinate or indeterminate 1D elements. Deflections on parts of beams (ends of beams or midspans) can't easily be evaluated, because the total deflection of a beam cannot be inferred from the deflection of a part of a beam.

Short-term $u_{z,st}$ or long-term $u_{z,lt}$ deflection is calculated and compared with the user-defined value $u_{z,lim}$.

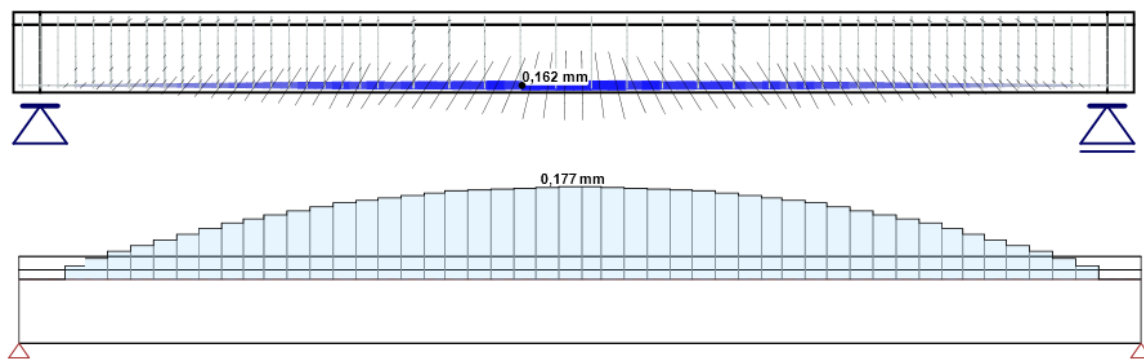


Fig. 7: Comparison of max. crack width between IDEA StatiCa Detail (above) and IDEA StatiCa RCS

Summary

The computational implementation of the compression field method enables a safe evaluation of discontinuity regions of concrete structures. It is a transparent method which provides the structural engineer with control over the behavior of the structure.

IDEA StatiCa Detail is a tool for the evaluation of discontinuity regions in concrete structures. It does not demand a deep understanding of issues concerned with the non-linear behavior of materials, nor does it require the setting up of a truss analogy model. It provides an effective way to assess details or walls and to evaluate and to interpret the results according to currently valid standards, including ULS and SLS evaluation.

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