Design of reinforced concrete sections according to EN 1992-1-1 and EN 1992-2

Educational text for practical seminar

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Foreword

Additionally to publication Design of reinforced concrete sections according to EN 1992-1-1 and EN 1992-2 – Validation examples is introduced theoretical part.

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Content

1. Ultimate limit state (ULS)

1.1. Bending with or without axial force

1.1.1. Methods for sectional capacity check

Two well-known methods can be used to check ultimate limit state. The first one will give us the cross sectional ultimate strength in the form of a interaction area or an interaction diagram (in the case of bending moment in one direction). Cross-sectional capacity can be determined as ratio of acting internal forces to limit state forces. The second one is finding equilibrium in cross section, where we are looking for the actual behaviour of the loaded section, the use of materials in terms of stresses and insight into the vulnerabilities of the section.

Both of these methods are based on assumptions which are outlined below.

1.1.2. General design assumptions

1. Strain ε in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis (plane sections remain plane).
2. Interaction of reinforcement and concrete is ensured by concrete and reinforcement compactness (strain ε supports the strain in concrete adjacent fibres are the same).

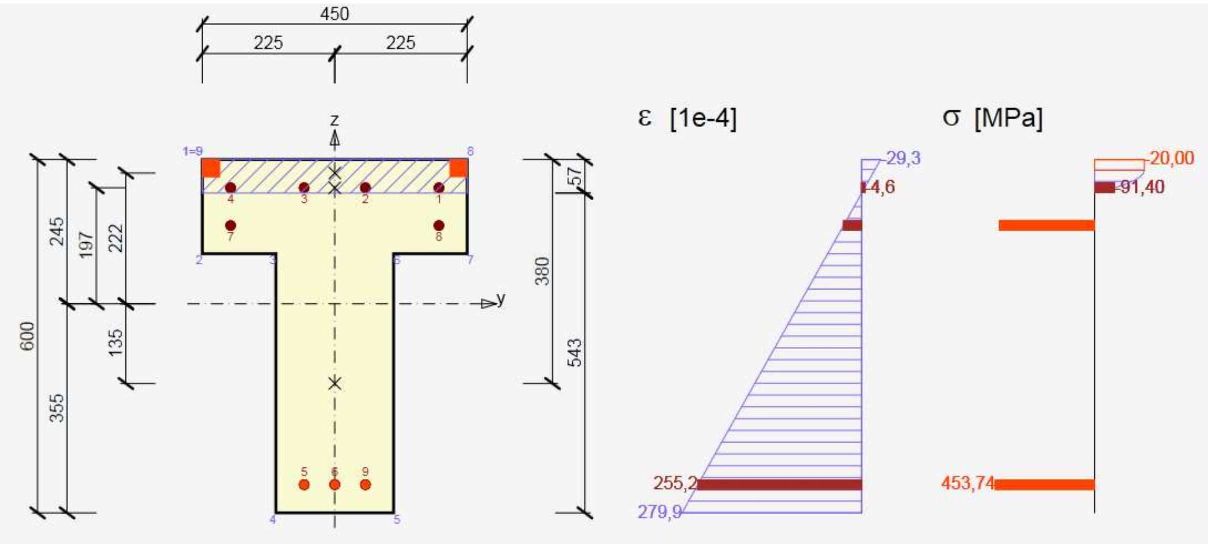


Fig. 1.1 – Strain stress

1.1.3. Calculation assumptions for Ultimate Limit State - ULS

1. Tensile strength of concrete is neglected (all tensile stresses are transmitted by reinforcement).
2. Concrete compression stresses in compression zone are calculated in relation to strain calculated from stress-strain diagrams.
3. Reinforcement stresses are calculated in relation to strain from stress-strain diagrams.

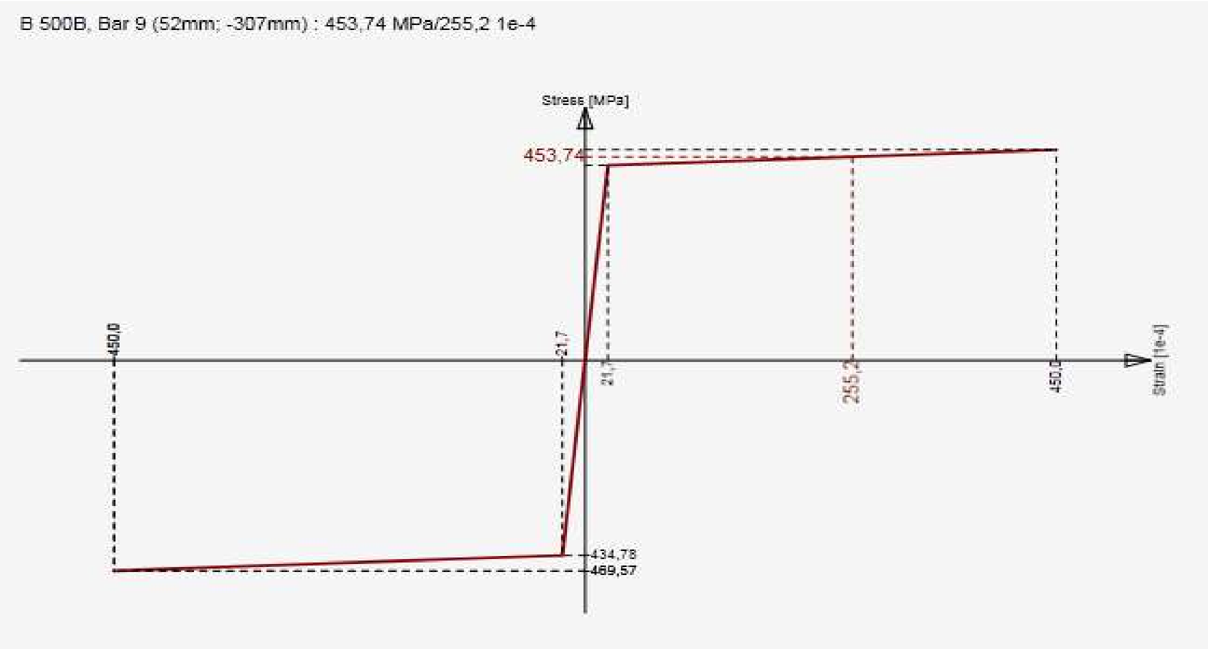


Figure 1. 2 –Stress-strain design diagram for reinforcing steel with inclined top branch

4. Compressive concrete strain with an ultimate strain limit ε_{cu2} (Parabola-rectangle diagram for concrete under compression) and ε_{cu3} (Bi-linear stress-strain relation), see tab. 3.1 and art. 6.1.7 [2].
5. Compressive strain of reinforcement is without limitation in case of horizontal plastic top branch, in case of inclined plastic top branch the strain is limited ε_{ud}, see art. 3.2.7 (2) [2].
6. As a limit state is considered the state when at least one of the materials exceeds the ultimate limit strain (if ε_u is not limited, the compressed concrete is governing).

1.1.4. Interaction diagram creating

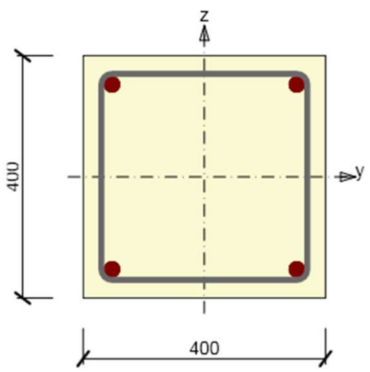


Fig. 1.3 – Reinforced cross-section

The first option is to check cross-section by interaction surface (interaction diagram). Explanation is provided on sample of the interaction surfaces for reinforced square section from the example in figure 1.3. On the interaction surfaces are located points defining ultimate limit state of examined cross-section. The interaction surface is drawn from the points (N, My, Mz), which are determined by stress integration in the cross section, which is achieved limit strain in one of the materials.
From 3D dimensional interaction surface can be derived 2D interaction diagram, which is a closed curve, which corresponds to the stress of constantly rotated neutral axis.

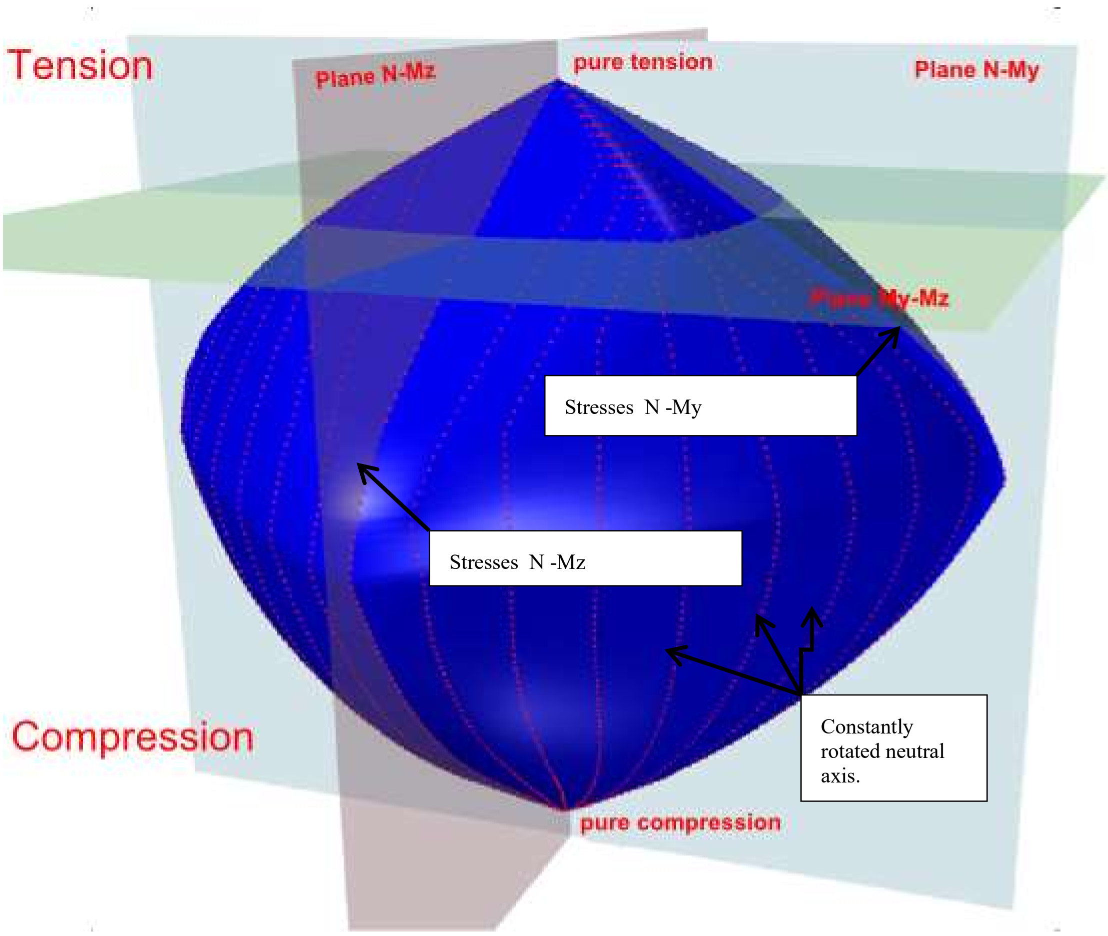


Fig. 1.4 – Interaction surface shows failure conditions for all load cases of normal force and bending moments

For case of symmetrical cross-section around y-axis the interaction diagram is symmetrical around plane N-M_y. Identically, for case of symmetrical cross-section around z-axis the interaction diagram is symmetrical around plane N-M_y. In case of cross-section reinforced by at on surface only we receive a flattened shape of interaction diagram.

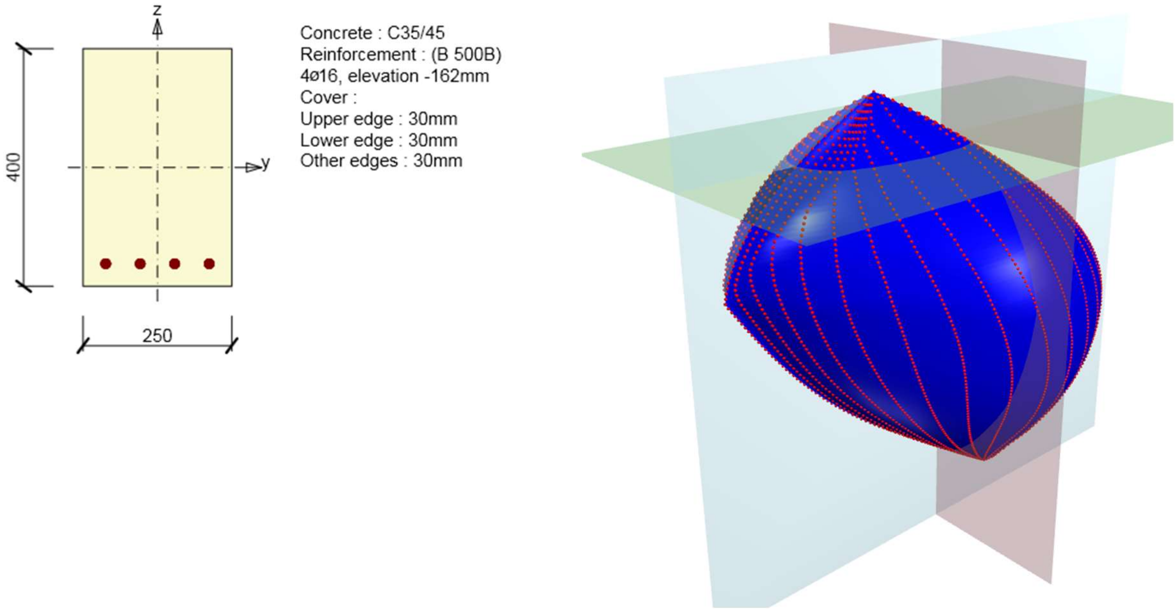


Fig. 1.5 – Interaction surface for cross-section with single symmetric reinforcement

As mentioned earlier, points defining ultimate limit state are received from stress integration. Fig. 1.6 displays Strain at the ultimate limit state.

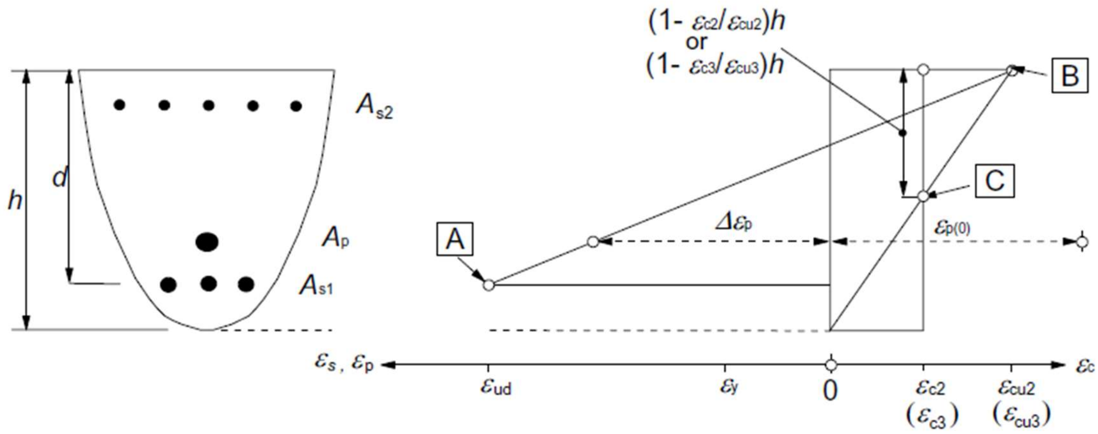


Fig. 1.6 - Strain distributions at the ultimate limit state (taken from [2])

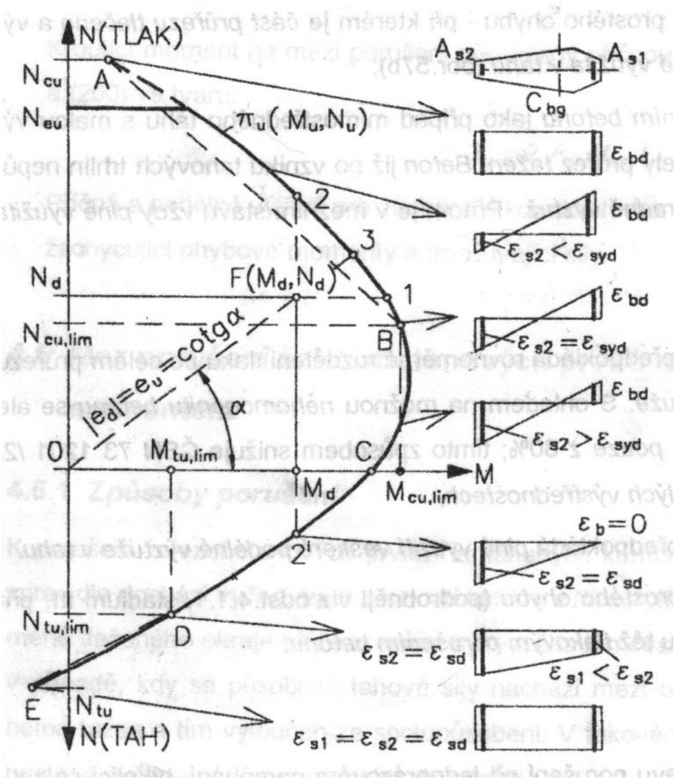


Fig. 1.7 – Interaction diagram shows cross-section failure under normal force and bending moments (taken from [1])

Respecting 2D diagram problem (closed curve laying on interaction surface) we can find out the strain plane is passing through neutral axis and critical point [y, z, ε], which is considered as critical point R. Point [y, z] defines point in cross-section with value of strain ε at the ultimate limit state. Neutral axis inclination is constant for all points of 2D diagram.

In case that the compressive stress in concrete is critical for design, the point R is matching to farthest compressed concrete fibre or to limiting point C – see Fig. 1.6. However, this can be applied only if that section is made from one type of concrete - not such as mixed cross-section.

In case that the tensile stress in reinforcement is critical for design (strain ε_{ud} is exceeded at the ultimate limit state for one or more bars), there must be fulfilled condition that for the given strain plane the value ε_{ud} is not exceeded at any other bar.

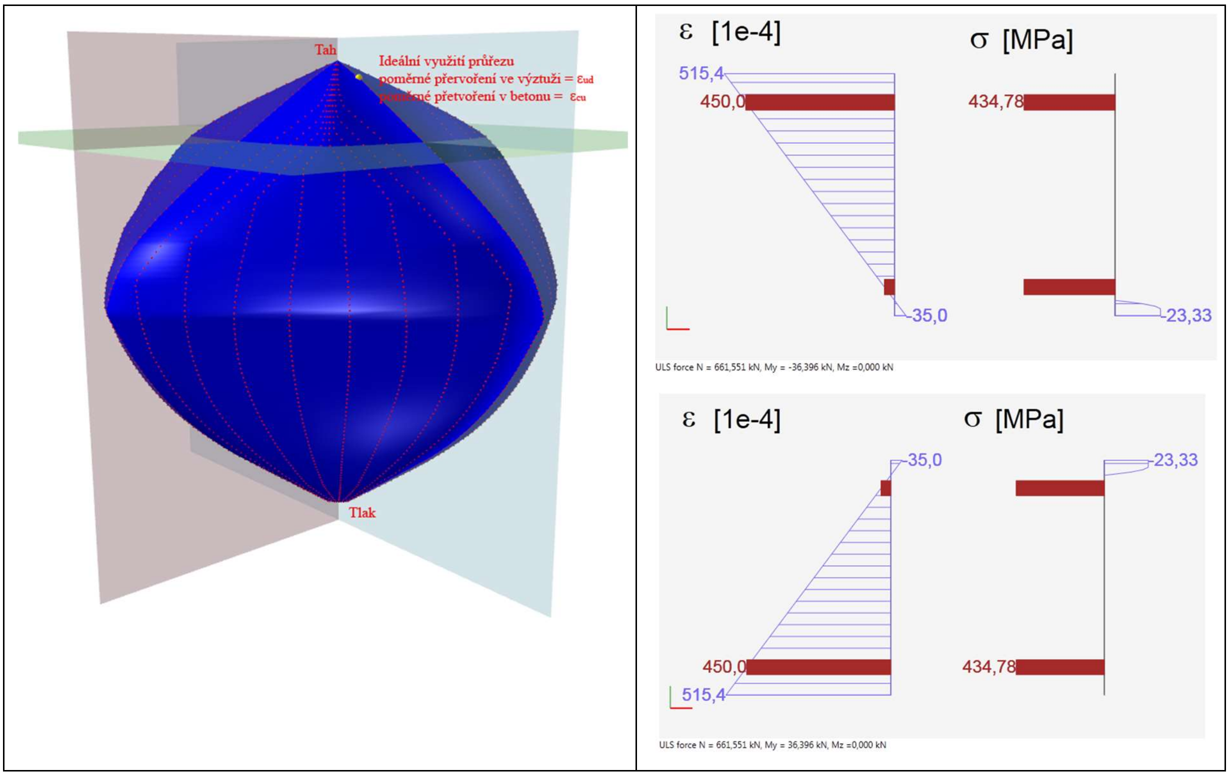
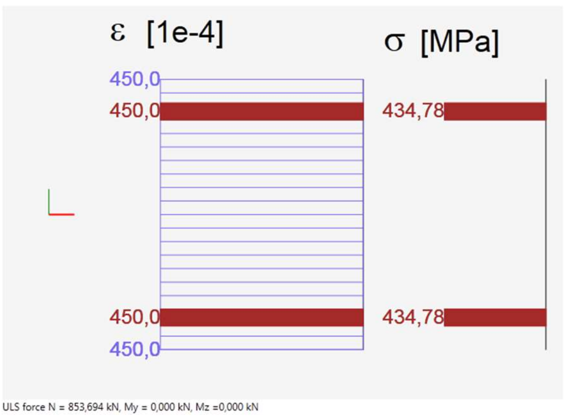
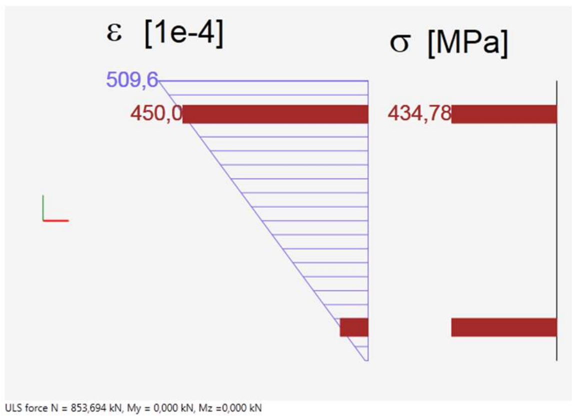


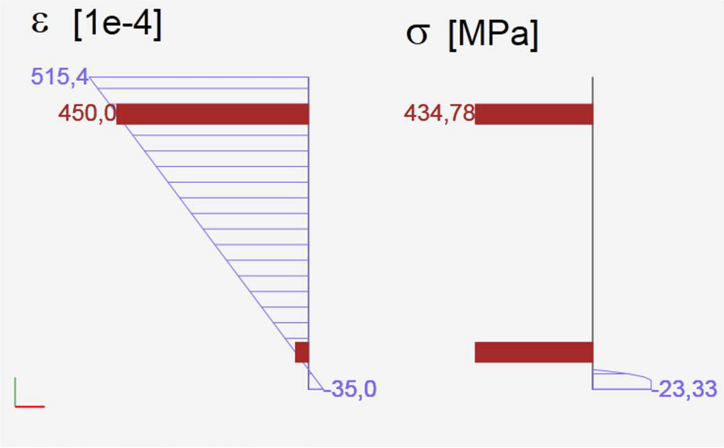
Fig. 1.8 – Optimal use of cross-section material



Point E – theoretical full use of tensile reinforcement

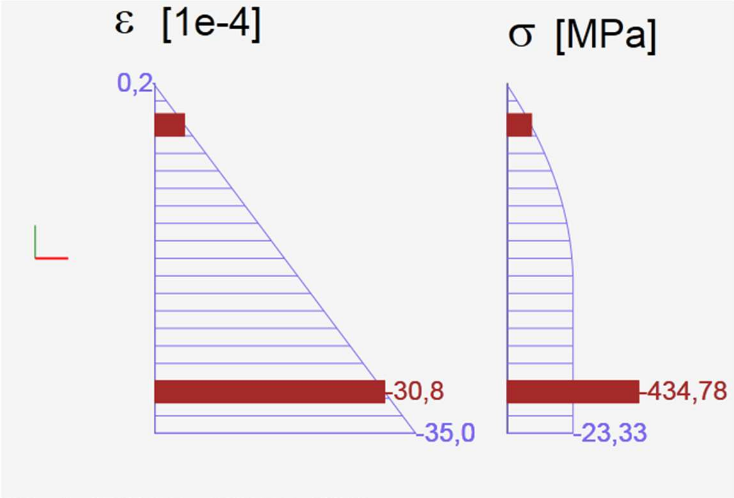


Point D



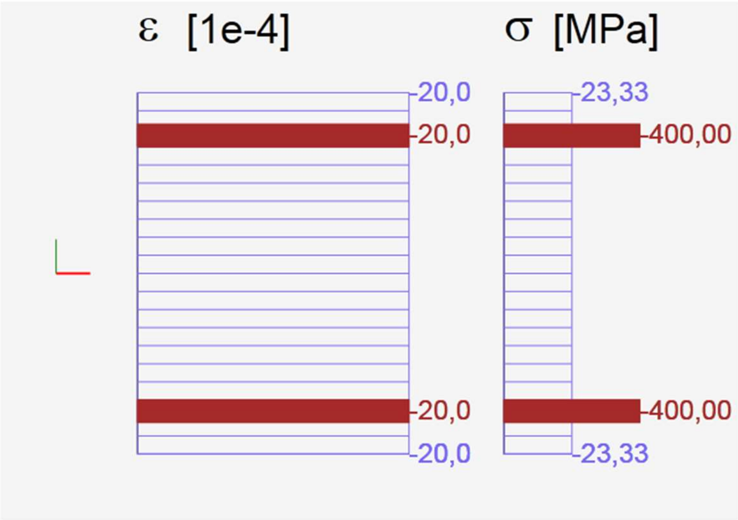
ULS force N = 661,551 kN, My = -36,396 kN, Mz =0,000 kN

Point B – Dividing case for failure by compressed concrete and tensile reinforcement (optimal use of cross-section materials)



ULS force N = -3506,832 kN, My = -157,415 kN, Mz =0,000 kN

The pivot point C (see - Figure in code)



ULS force N = -4518,731 kN, My = 0,000 kN, Mz =0,000 kN

Point A – theoretical full use of compressed concrete and reinforcement

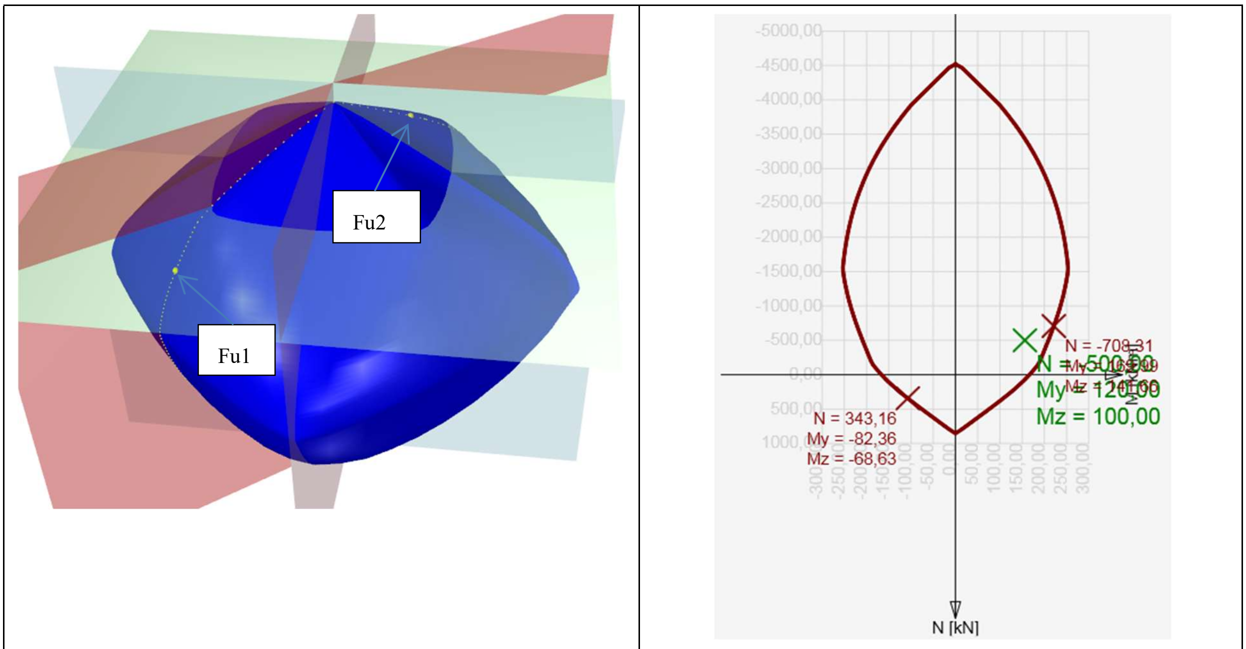
Fig. 1.9 – Characteristic strain plane positions calculated for purpose of interaction diagram (computed by program IDEA RCS)

The picture above shows that the diagram can be divided into two parts: the part where the failure is caused by tensile force and the part which failures by compressed force. Boundary points correspond to the case of Figure 1.9, where the extreme inclination of the strain plane also can be seen. When drawing an interaction diagram the plane strain inclination of cross section is changing in this interval, while we search the point R, see above. Based on that defined plane we figure out the integration to get the stress at the ultimate limit state.

1.1.5. Cross-section check subjected to axial force and bending moment

The check of cross-section subjected to axial force and bending moment inheres in proving that checked stresses (combination N_d, M_{zd}, M_{zd}) are located inside or on the surface interaction area. This can be done by different methods. The following example demonstrates the check of our rectangular cross-section subjected to forces $N_d = -500$ kN, $M_{yd} = 120$ kNm, $M_{zd} = 100$ kNm.

1.1.5.1. Method NuMuMu

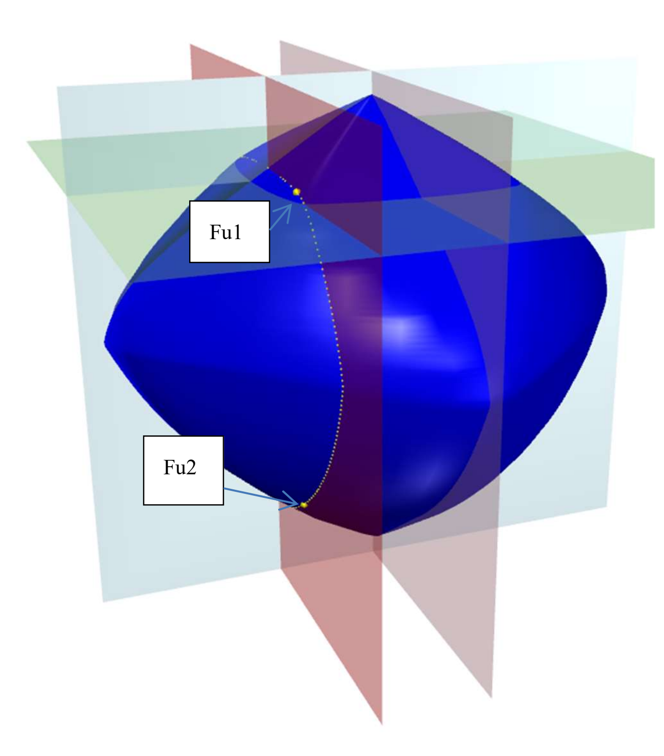


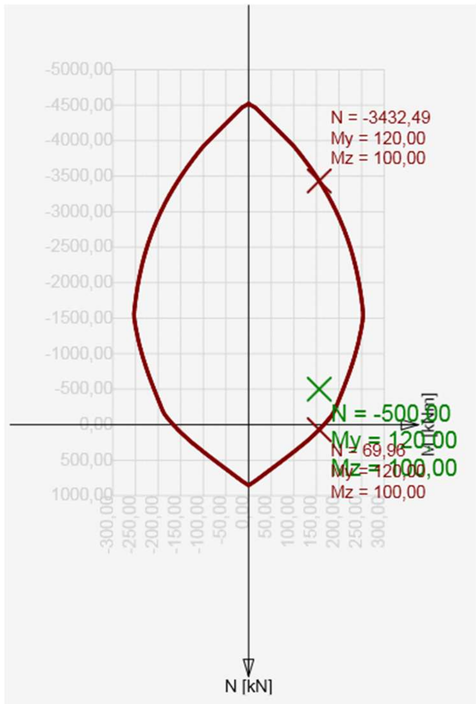
Section of diagram (Myd - Mzd)

To define resistance of cross-section we assume proportional changes in all internal forces components (the eccentricity of the normal force remains constant) until the achievement of interactive surfaces. The change of involved internal forces can be interpreted as moving along a line connecting the start coordinate system (0,0,0) and the point defined by the internal forces (N_{Ed}, $M_{Ed,y}$, $M_{Ed,z}$). The two intersections of this line with the interaction surface, which can be found, represent two sets of forces at the ultimate limit state.
At each intersection the program determines three forces at the limit state: the design axial force resistance N_{Rd} and the corresponding design moment resistance M_{Rdy}, M_{Rdz}.

1.1.5.2. Method NuMM

To define resistance of cross-section we assume constant bending moments (which is equal to the active design moments) and a gradual changes in normal force until the achievement of interactive surface. The change of involved internal forces can be interpreted as moving in vertical plane along the line connecting the point (0, $M_{Ed,y}$, $M_{Ed,z}$) and the point defined by the acting internal forces (N_{Ed}, $M_{Ed,y}$, $M_{Ed,z}$). The two intersections of this line with the interaction surface, which can be found, represent two sets of forces at the ultimate limit state. At each intersection the program determines three forces at the limit state: the design axial resisting force N_{Rd} and (corresponding) acting design moments $M_{Ed,y}$ a $M_{Ed,z}$.

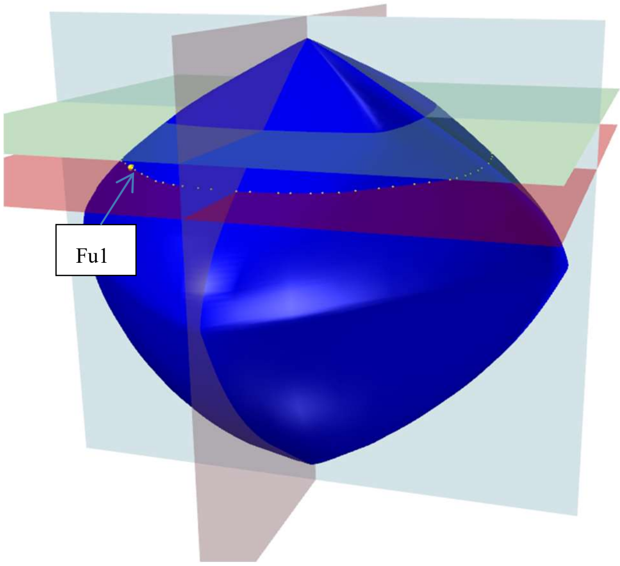


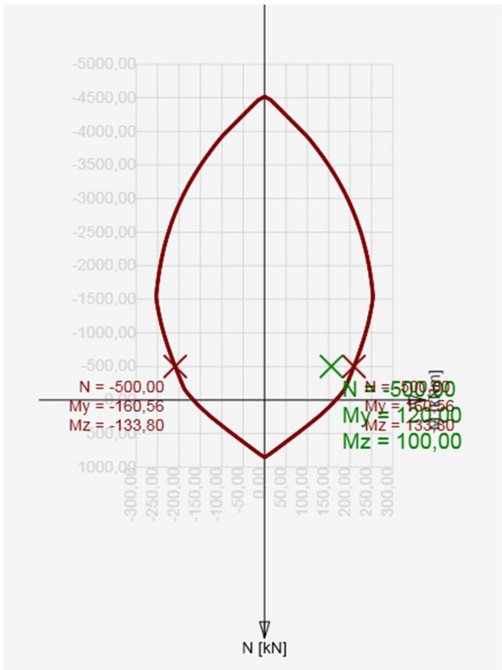


Section of diagram (Myd - Mzd)

1.1.5.3. Method NMuMu

To define resistance of cross-section we assume constant normal force (which is equal to the acting design normal force) and proportional changes in bending moments until the achievement of interactive surface. The change of involved internal forces can be interpreted as moving in horizontal plane along the line connecting the point (N_{Ed},0,0) and the point defined by the acting internal forces (N_{Ed}, $M_{Ed,y}$, $M_{Ed,z}$). The two intersections of this line with the interaction surface, which can be found, represent two sets of forces at the ultimate limit state. At each intersection the program determines three forces at the limit state: the design resisting moments M_{Rdy}, M_{Rdz} and (corresponding) acting design normal force N_{Ed}.





Section of diagram (Myd - Mzd)

1.1.1. Finding section response

Another possibility to check cross-section is through finding cross-section response (i.e. Strain and stress distribution from acting internal forces). This method is also known as a method of limit deformation. The level of acting stresses in each fibre (in the case of plane bending in each layer) in each reinforced bar is calculated depending on the strain of the Stress-strain diagram of the material.
Finding the cross-section response is figure out using numerical method specified in [6]. The principle consists in the gradual load increment of the section by the unbalanced components of a not-transferred forces. Those are obtained by integrating the stress over the section using Stress-strain diagrams. If the stress value can be found for the strain in the Stress-strain diagram, see Figure 1.10 (a), the calculated stress is correct assuming linear elastic material. In cases (b) and(c), the stress for a
linear calculation reaches unrealistic values, and part (b) or entire value (c) cannot be transmitted by material.

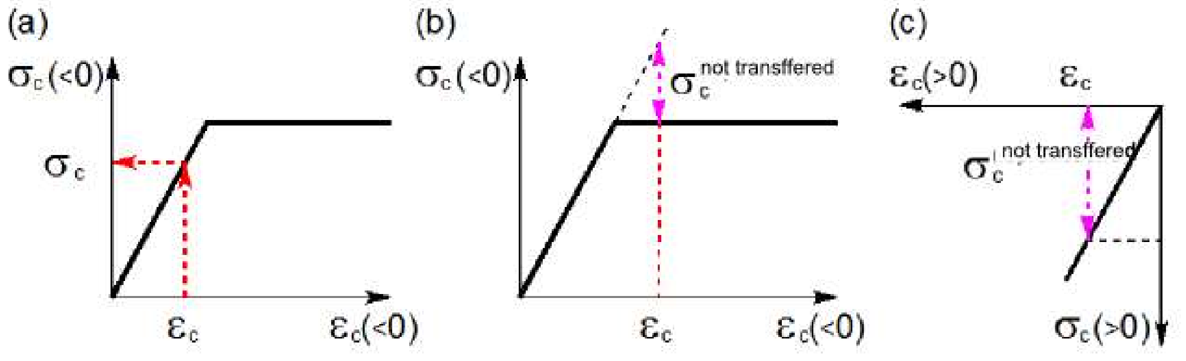


Fig. 1.10 – Not-transferred stresses in Stress-strain diagrams [4]

Integrating not-transferred stresses we get not-transferred internal forces and their resultants should be added to the internal forces of variable loads, see Figure 1.11

Fig. 1.11 – Not-transferred inner forces [4]

This calculation method requires the use of numerical methods for integrating the stress over the cross section area and for nonlinear analysis equilibrium equations in the section. Iteration is terminated at the time when the convergence criteria are met.

$$\frac{F_e - F_i}{F_e} \leq max\{\varepsilon, \delta\}$$

where F_e is section load,

F_i is section response (internal forces calculated on base of strain plane).

If a is approximate (approximated) value and a b is exact (true) value, then absolute deviation is given by following equation

$$\varepsilon = |b - a|$$

Relative deviation is given by following formula:

$$\delta = \left|\frac{b - a}{b}\right|$$

In most programs, you can set these convergence criteria (default values are 1% as relative error, 100 N, 100 Nm as the absolute error of normal force and moments). So if we have the input of N = 0 kN, My = 100 kNm, Mz = 0 kNm and integrated forces after iteration N = - 0.07 kN, My = 100,5 kNm, Mz = 0.02 kNm, the evaluation will be as follows.

With respecting the N and Mz are equal to 0, comparison with absolute deviation can be done, which is satisfying in our case

The value of normal force 100N> | 70 | N
The value of the bending moment Mz 100Nm> | 20 | Nm
The value of the bending moment My

$$\delta = \left|\frac{b-a}{b}\right| = \frac{100-100{,}5}{100} = 0{,}005 \ < 0{,}01$$

With respect to satisfying comparison to relative deviation, the comparison to absolute deviation is not needed.

1.1.2. Cross-section check by response

In the case of finding a balance in cross section, plane strain is known. From the plane strain we can calculate strain anywhere in section, then the stress or inner forces in reinforcement bars, cross-section or its parts using Stress-strain diagrams of the materials. The calculated stress and strain values we compare with the limit strain value from stress-strain diagrams of used materials.

The advantage of this method is that we get a complete image about the stress and strain values in the section of the internal forces acting on the cross-section.

1.2. Shear

With respect to fragile failure the Shear check is one of the important checks of reinforced concrete section.

1.2.1. Calculation procedure

Calculation of shear resistance is composed of several basic parts. At first we should analyze whether the bending cracks occur or not in checked location. If any, use the calculation according to EN 1992-1-1[2], Article 6.2.2 (1). Otherwise, we determine whether it is plane concrete or poorly reinforced concrete, then proceed in accordance with EN 1992-1-1 Article 12.6.3.

For reinforced uncracked concrete (without shear reinforcement) we check according to EN 1992-1-1 Article 6.2.2 (2).

For Elements, where is required shear reinforcement we check according to Article 6.2.3 [2].

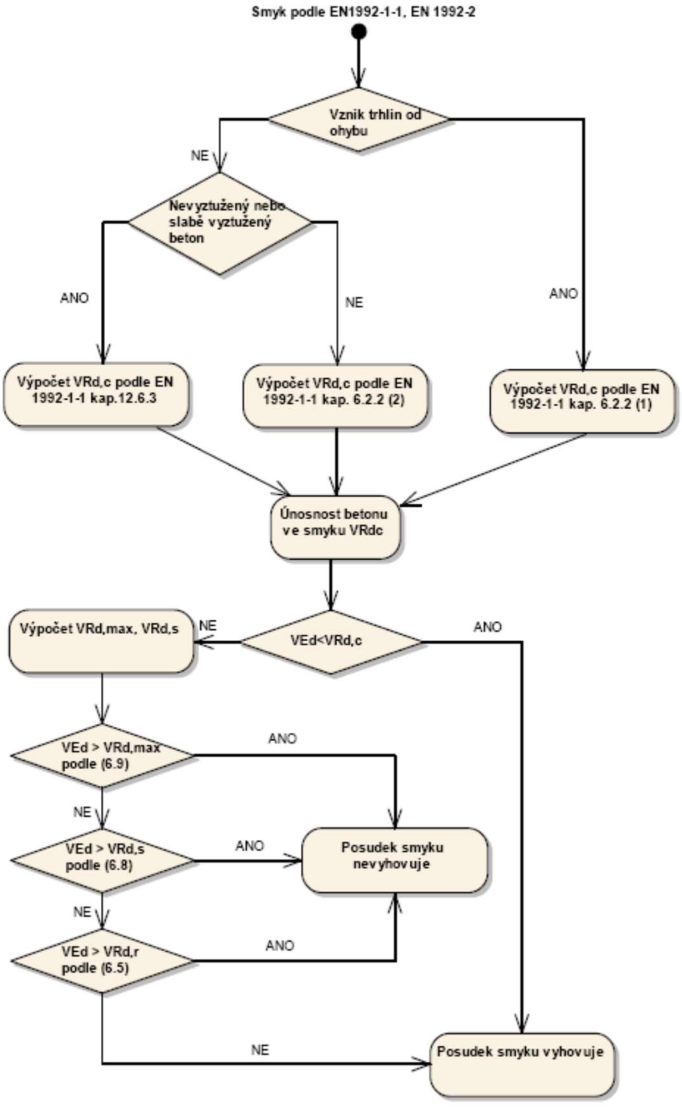


Fig. 1.12 - Process diagram for shear check

1.2.2. Shear resistance of members without shear reinforcement

1.2.2.1. Shear resistance of members in cracked bending zones (art. 6.2.2 (1) [2])

Shear resistance of reinforced concrete members without shear reinforcement subject to bending moment is given by:

$$V_{Rd,cm} = C_{Rd.c} k\,(100\,\varrho_l f_{ck})^{1/3}\,b_w d,$$

Which was defined on the base of tests executed on representative number of simple beams in case of failure by shear force. Since the above resistance may be zero for elements without longitudinal reinforcement (ρ l), for poorly reinforced members was derived equations. Since the above resistance may be zero for members without longitudinal reinforcement (ρ_l), for the poorly reinforced members was determined equation.

$$V_{Rd,c} \geq \upsilon_{min} b_w d.$$

For shear resistance with influence of normal force was determined equation

$$V_{Rd,cn} = k_1 \sigma_{cp}\,b_w d$$

Shear resistance in its complete expression which is corresponding with EN 1992-1-1 art. 6.2.2 (1)

$$V_{Rd,c} = \left[C_{Rd.c} k\,(100\,\varrho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\right]\,b_w d$$

With minimum of

$$V_{Rd,c} = (\upsilon_{min} + k_1 \sigma_{cp}) b_w d$$

where $C_{Rd,c}$ = 0,18 / γ_c,

k cross-section height factor $k = 1 + \sqrt{\frac{200}{d}} < 2{,}0;$ with d in mm,

ρ_l reinforcement ratio for longitudinal reinforcement $\varrho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02,$

f_{ck} characteristic compressive cylinder strength of concrete at 28 days,

k_1 = 0,15,

σ_{cp} = N_{Ed} / A_c < 0,2 f_{cd} v MPa,

b_w smallest width of the cross-section in the tensile area,

d effective depth of a cross-section, see 1.2.4.2,

ν_{min} minimal equivalent shear strength $\nu_{min} = 0.035\, k^{3/2}\, fck^{1/2}$.

1.2.2.2. Shear resistance of members in cracked bending zones (art. 6.2.2 (2) [2])

Shear resistance of members in uncracked bending zones can be determined from Mohr circle. Into equation

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_z^2}$$

We substitute $\sigma_x = \sigma_{cp}$ a $\tau_z = V_{Rd,c}\,S / (I\,b_w)$ and figure out $V_{Rd,c}$ and get equation corresponding with formula given in EN 1992-1-1 art. 6.2.2 (2)

$$V_{Rd,c} = \frac{I\, b_w}{S}\sqrt{f_{ctd}^2 + \alpha_l \sigma_{cp} f_{ctd}}$$

where I is the second moment of area,
b_w is the width of the cross-section at the centroidal axis
S is the first moment of area above and about the centroidal axis,
f_{ctd} design axial tensile strength of concrete in MPa,
σ_{cp} is the concrete compressive stress at the centroidal axis due to axial loading and/or prestressing,
α_l transmission length factor, usually 1,0.

In relation with the above it should be noted that in areas without bending cracks the resistance $V_{Rd,c}$ can be significantly higher than in cracked areas according to Article 6.2.2 (1) [2], see Figure 1.13. This figure clearly shows that although the shear force is checked at its extreme (which does not produce cracks), need not necessarily ensure that it will be transferred along the whole beam length. It is due to a change in the method of calculating the shear resistance of the concrete. On the safe side, of course, the shear resistance can be considered according to Article 6.2.2 (1) [2] also in places where cracks will not occur.

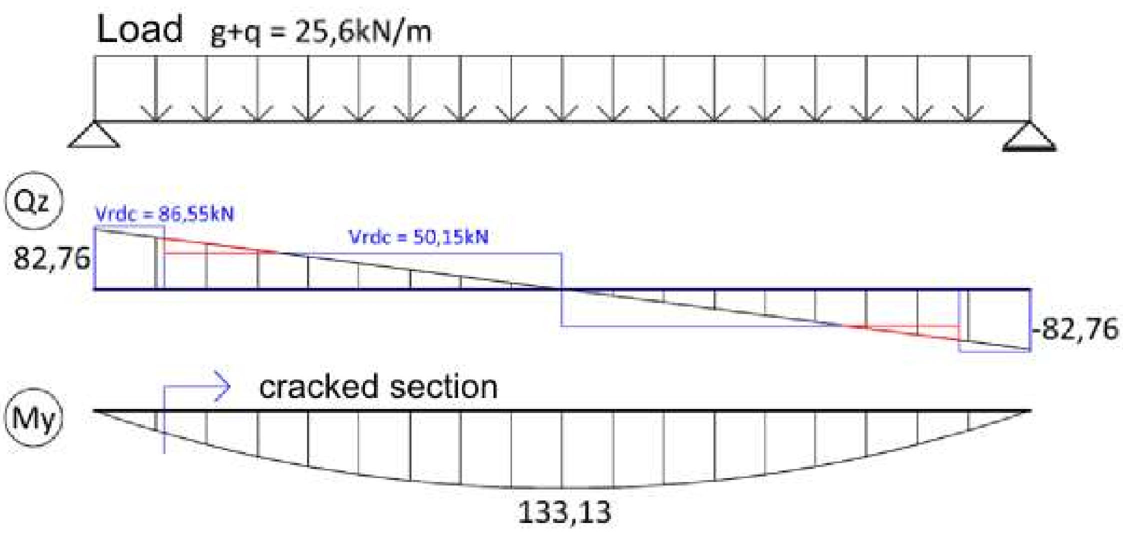


Fig. 1.13 – Shear resistance comparison before and after the cracks occurred

To the expression of $V_{Rd,c}$ according to Article 6.2.2 (2) must also be noted that in the general case should be based on check at the fibre of the extreme principal concrete tensile stress in zone of normal compressive stress, but not at the centre of gravity of the section. At this point it is necessary to calculate the cross-sectional characteristics (S and b_W). To determine the maximum principal stress σ_1 in program IDEA RCS we draw a line through the centre of gravity in the direction of the resultant shear forces. This line we divide to 20 sectors. On this line we will present more characteristic points (points of the cross-section polygon, centre of gravity, the neutral axis). Within these points, we calculate S, b_w, σ_x, τ_{yz} a σ_1. At the point of maximum principal tensile stress we will calculate the shear resistance.

Shear force before applying the reduction factor β required by Article 6.2.2 (6) must satisfy the extra condition

$$V_{Ed} \leq 0{,}5\, b_w d\, \upsilon\, f_{cd}$$

where ν $= 0{,}6\left[1 - \frac{f_{ck}}{250}\right]$ kde f_{ck} je v MPa,

1.2.2.3. Shear resistance of members without reinforcement or lightly reinforced (art. 12.6.3 [2])

Shear resistance for plain or lightly reinforced concrete can be determined from formula:

$$\tau_{cp} \leq k\, V_{Ed}\, / A_{cc},$$

Where τ_{cp} we substitute by

$$f_{cvd} = \sqrt{f_{ctd,pl}^2 + \sigma_{cp} f_{ctd,pl}}\ pro\ \sigma_{cp} \leq \sigma_{c,lim}$$

or

$$f_{cvd} = \sqrt{f_{ctd,pl}^2 + \sigma_{cp} f_{ctd,pl} - \left(\frac{\sigma_{cp} - \sigma_{c,lim}}{2}\right)^2}\ pro\ \sigma_{cp} > \sigma_{c,lim}\ .$$

Partial values used above formula are given by:

$$\sigma_{cp} = \frac{N_{Ed}}{A_{cc}}$$

$$\sigma_{c,lim} = f_{cd,pl} - 2\sqrt{f_{ctd,pl}\left(f_{ctd,pl} + f_{cd,pl}\right)}\ ,$$

where $f_{cd,pl}$ Design compressive strength for plain or lightly reinforced concrete,
$f_{ctd,pl}$ Design axial tensile strength of plain or lightly reinforced concrete,
f_{cvd} Design shear resistance under concrete compression.

1.2.3. The resistance of members with shear reinforcement (art. 6.2.3 [2])

Calculation of reinforced concrete members resistance with shear reinforcement is based on the truss analogy method with variable-angle diagonals. The basis of this method is the balance of forces in the triangle determined by the strut force (diagonal), the shear reinforcement force (stirrup) and longitudinal reinforcement force.

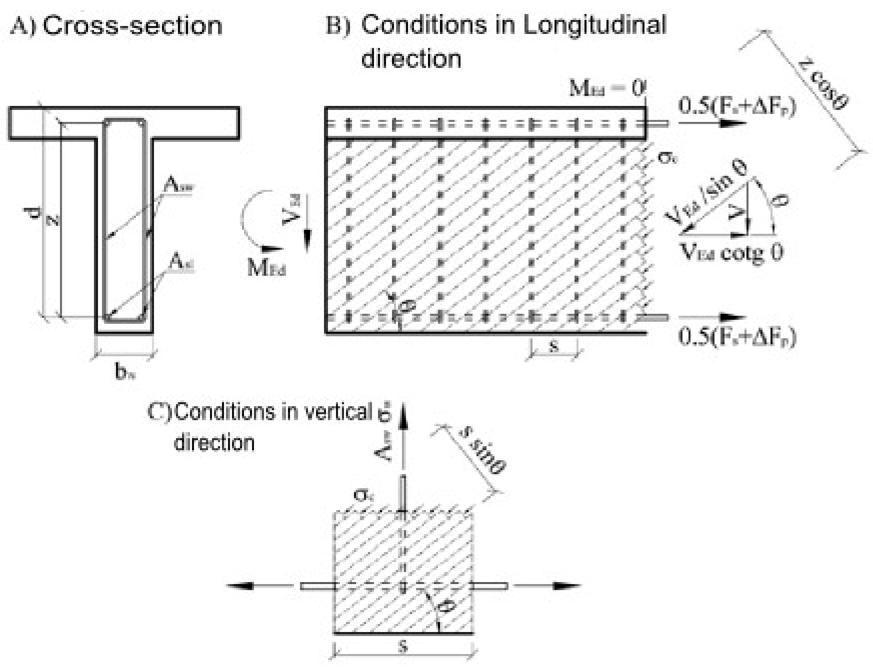


Fig. 1.14 – Principe of Truss analogy for member under shear load

Cross-section under shear load is broken by cracks at an angle θ, from this reason the concrete diagonal with same angle as shear forces is resisting to the shear force. Compressive force of the diagonal can be expressed as $V_{Ed}\ /\sin\theta$. This force must be transferred by concrete surface, perpendicular to the compression diagonal $b_w z\ \cos\theta$. The concrete tension stress in the compression diagonal is then equal

$$\sigma_c = \frac{V_{Ed}}{b_w z\ \sin\theta\cos\theta} = \frac{V_{Ed}}{b_w z}(\tan\theta + \cot\theta)$$

Substituting $\sigma_c = \alpha_{cw}\nu_1 f_{cd}$ a $V_{Ed} = V_{Rd,max}$ and expressing $V_{Rd,max}$ we get equation for shear resistance of diagonal

$$V_{Rd,max} = \ \alpha_{cw}\ b_w\ z\ \nu_1\ f_{cd}/(\cot\theta + \tan\theta).$$

To balance the vertical force component in compression diagonal the shear reinforcement will be used. The size of the vertical force is based on the diagonal compressive stress in the concrete area which is corresponding to one single stirrup - $\sigma_c b_w s\sin^2\theta.$ Limit stirrup force is given as$A_{sw} f_{ywd}/s$.

Inserting σ_c, comparing with the limit force in the reinforcement, after modifications we get

$$\frac{A_{sw} f_{ywd}}{s} = \frac{V_{Ed}}{z}\tan\theta$$

Then expressing V_{Ed} as V_{Rds} we get resistance of cross-section with vertical shear reinforcement

$$V_{Rd,s} = \frac{A_{sw}}{s} z\ f_{ywd}\cot\theta.$$

The longitudinal shear force is transferred by longitudinal reinforcement and it can be determined as $V_{Ed}\cot\theta$. Derivation of formulas above can be found in [4].

By program IDEA RCS is possible to check only members with vertical shear reinforcement. In general following equations can be used:

$$V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd}\,(\cot\theta + \cot\alpha)\sin\alpha$$

$$V_{Rd,max} = \alpha_{cw}\, b_w\, z\, \nu_1\, f_{cd}(\cot\theta + \cot\alpha)/(1 + \cot^2\theta)$$

Where A_{sw} is the cross-sectional area of the shear reinforcement,

s is the spacing of the stirrups,

f_{ywd} is the design yield strength of the shear reinforcement,

b_w is the minimum width between tension and compression chords. For the calculation of resistance $V_{Rd,max}$ is needed to reduce this value to nominal cross-section width in case, that cross-section is weakened by ducts

$$b_{w,nom} = b_w - 0{,}5\ \Sigma\phi\ For \text{ grouted metal ducts,}$$

$$b_{w,nom} = b_w - 1{,}2\ \Sigma\phi \text{ For non} - \text{grouted ducts,}$$

ν = 0,6 pro $f_{ck} \leq$ 60MPa or $0{,}9 - f_{ck}/200$ pro $f_{ck} >$ 60MPa,

α_{cw} is a coefficient taking account of the state of the stress in the compression chord.

Load	**$\sigma_{cp} = 0$**	**$0 < \sigma_{cp} \leq 0{,}25\ f_{cd}$**	**$0{,}25\ f_{cd} < \sigma_{cp} \leq 0{,}5\ f_{cd}$**	**$0{,}5\ f_{cd} < \sigma_{cp} \leq 1{,}0\ f_{cd}$**
Coefficient α_{cw}	1.0	$1+\sigma_{cp}/f_{cd}$	1,25	$2{,}5(1 - \sigma_{cp}/f_{cd})$

Tab. 1-1 Determining coefficient α_{cw}

Angle θ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force. The limiting values of $\cot\theta$ for use in a Country may be found in its National Annex. The recommended limits are given by expression $1 \leq \cot\theta \leq 2{,}5$.

Choosing the size of the angle θ can affect the value of the resistances. Dependence of resistances is visible in Figure 1.15. The figure shows that with increasing of angle θ the resistance $V_{Rd,max}$ is increasing, and resistance $V_{Rd,s}$ is decreasing. Resistance $V_{Rd,c}$ is constant, since it is based on the truss analogy method.

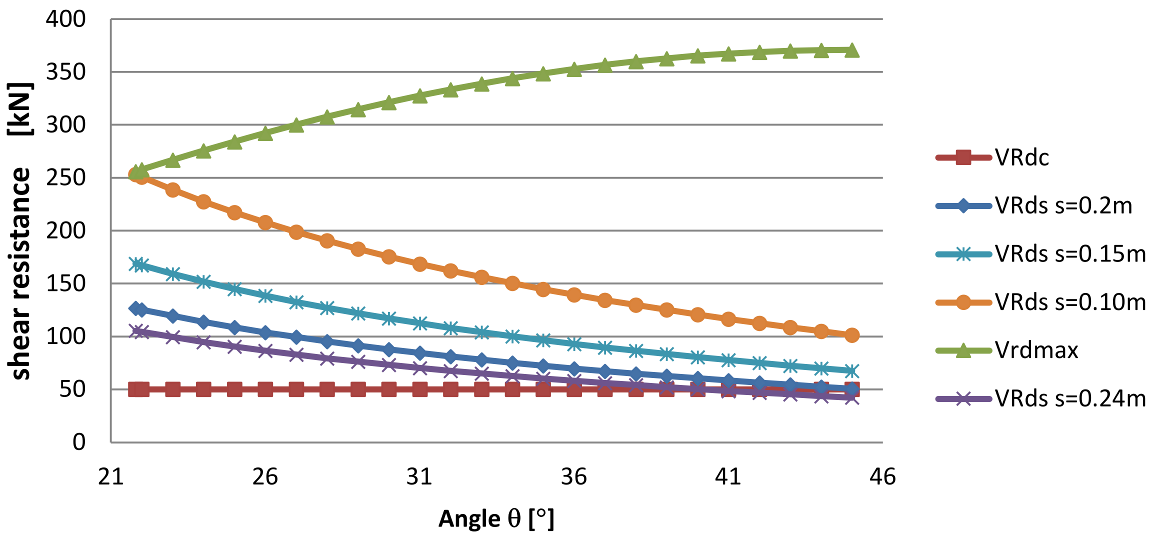


Fig. 1.15 – Dependency between shear resistance and angle θ

1.2.4. Cross-section characteristics calculation for shear

To calculate the shear is important to calculate the cross-sectional variables affecting the shear resistance. These variables are particularly resistant shear section width b_w, the effective width d and lever arm z. The code [2] gives these values directly correlated with the actual bending stress. But the problem is to determine these values when the direction of the resultant bending moments (or more accurately the direction of the resultant of section resistance) is significantly different from the direction of the resultant shear forces. In this case, the EC2 code doesn't provide any recommendations.

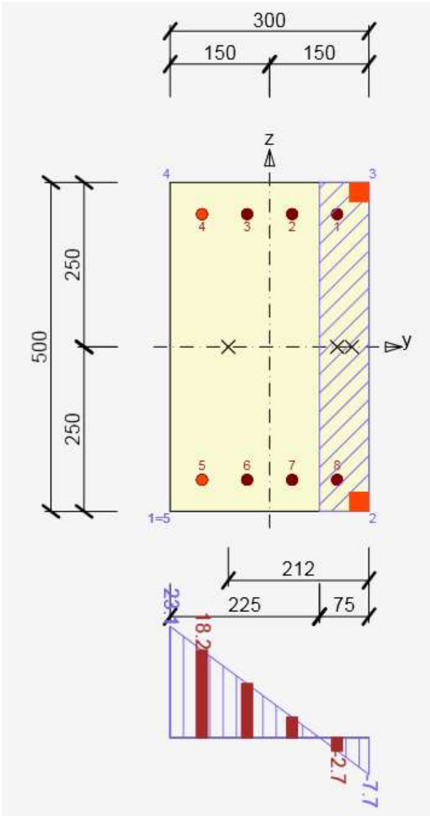


Fig. 1.16 – Bending acting perpendicularly to shear force

1.2.4.1. Cross-section width resisting to shear b_w

In the IDEA RCS program calculates the cross-section width resisting to shear in the direction perpendicular to the resultant of shear forces. Depending on the article in the Eurocode this width is calculated as:

- The smallest width of the section between the resultant of compressive concrete and tensile reinforcement in the direction perpendicular to the resultant of shear forces for article 6.2.2 (a) and 6.2.3 (1)
- The section width in a direction perpendicular to the resultant of shear forces in the checked point according to article 6.2.2 (2)

1.2.4.2. Effective depth of a cross-section

Effective depth is usually defined as the distance of most compressed concrete fiber to the centre of gravity of reinforcements. Because it is directly related to the bending, the distance is given as perpendicular projection to the gravity line of the plane strain.

This definition can be clarified so that instead of centre of gravity of the tensile reinforcement is used the position of the reinforcement resultant of forces. During the development the IDEA RCS program the problem was solved, how to define the effective depth of the cross-section, for which the plane of bending loads doesn't correspond with the direction of the resultant shear forces. Therefore, the effective depth is defined as the distance of most compressed concrete fibre to the resultant forces in the tensile reinforcement (based on bending stress) and in the direction of the resultant shear forces, see Figure 1.17.

Exceptional cases will occur if we are not able to determine the compressed fiber or resultant in the tensile reinforcement. In this case, we recommend using value 0.9 h (90% of section depth in the direction of the resultant shear forces). This value, the user can define in the IDEA RCS program by setting of code variables

1.2.4.1. Lever arm of internal forces

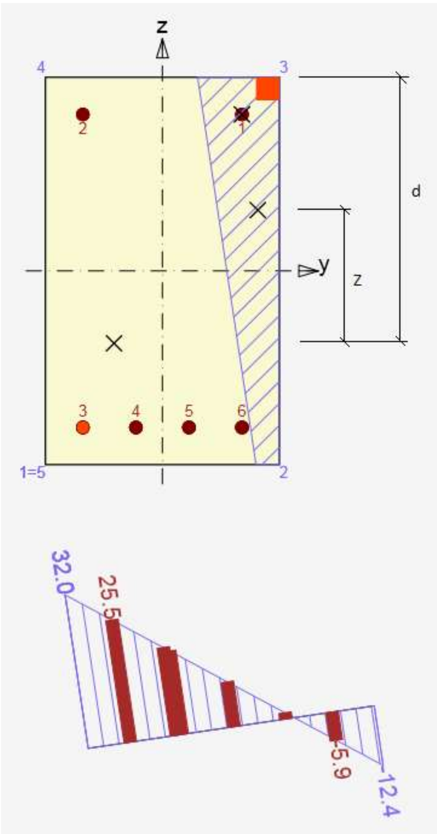


Fig. 1.17 – Principle of defining effective depth and lever arm for shear check

The lever arm of internal forces is in 6.2.3 (3) [2] defined as "distance between tension and compression chords". The code does not define how to proceed when the plane of acting bending moment is different from direction of the resultant shear forces. Therefore, as for the case of the effective depth, we define the distance in the direction of the resultant shear forces. Also here, we can face a similar exception cases, for example, the whole section is under compression, etc. In this case, we take value 0.9 d (90% of the effective section height). This value, the user can set in the IDEA RCS program by setting of code variables.

Dependence between bending plane inclination and the resultant of shear force is clearly visible in Figure 1.18 and Figure 1.19.
With increase of inclination the values of effective height, lever arms and related resistances are decreasing. The limit state is 90°.
For this inclination the lever arm of internal forces cannot be calculated, consequently the lever arm is equal to zero. In this case, is considered the value specified in the setting of code variables. By this, there is a jump at the end of the chart. This study proves the recommended maximum for inclination about 20°.

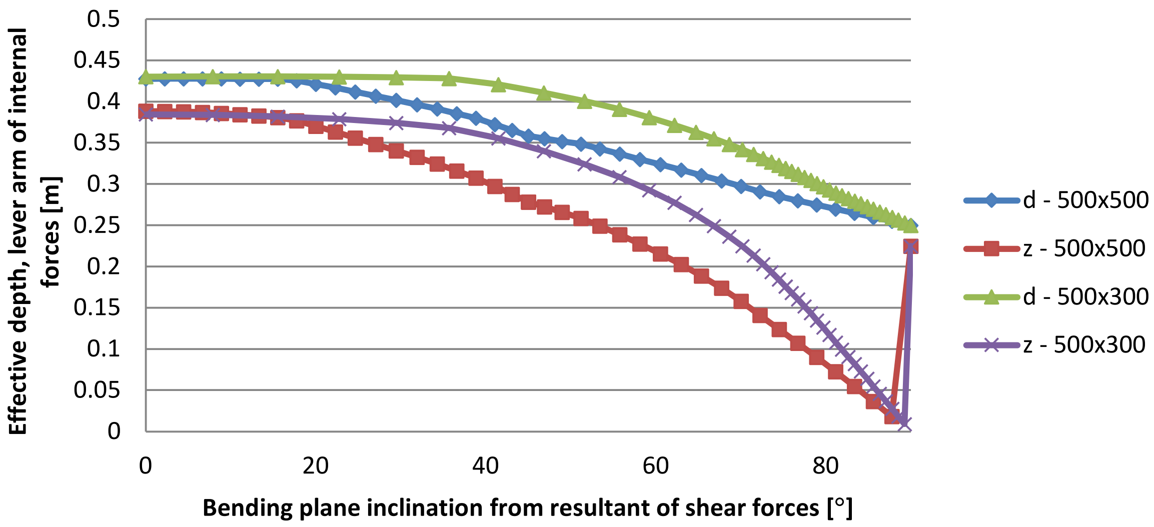


Fig. 1.18 - Dependence between effective depth, lever arm to the bending plane inclination and the resultant of shear forces

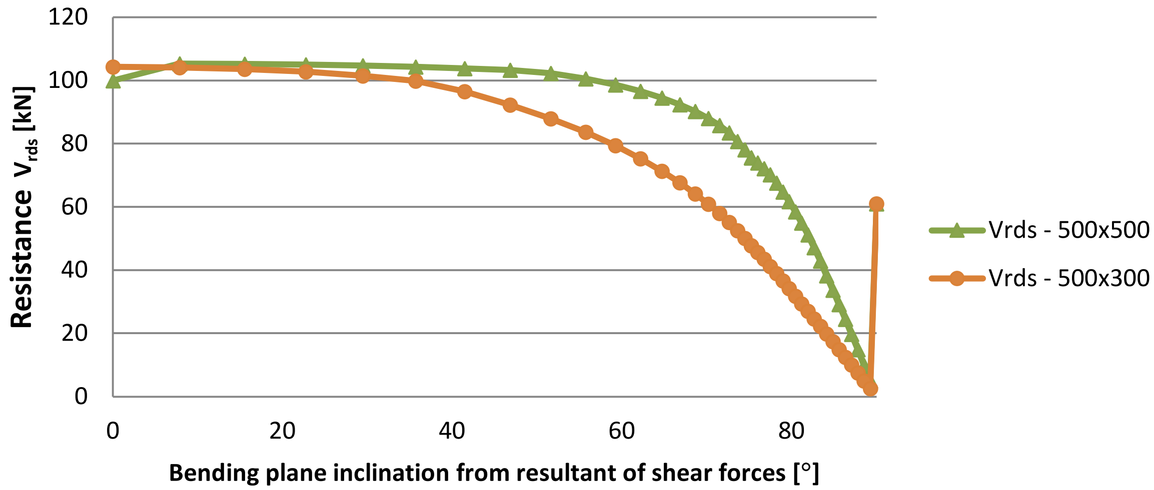


Fig. 1.19 - Dependence between resistance V_{rds} to the bending plane inclination and the resultant of shear

During testing the RCS program, the study about dependency of shear resistance to changing the normal force was proceeded. Resistance $V_{Rd,max}$ is affected only by the coefficient α_{cw}, see Fig. 1.20. Fig. 1.21 shows a constant value of resistance V_{Rds}.
For V_{Rdc} resistance, the decreases cause increasing of normal force. The blue curve in Fig. 1.21 shows the resistance V_{Rdc} with neglecting the influence of cracks and it was calculated using the formula in section 6.2.2 (1) [2]. Jump in transition between pressure and tension is caused by contributed tensile reinforcement. Red curve respects the influence of crack and till the moment of first crack from bending the resistance decreases. It is calculated using the formula in section 6.2.2 (2) [2]. After the first crack occurred the dependency curve is same as for 6.2.2 (1) [2].

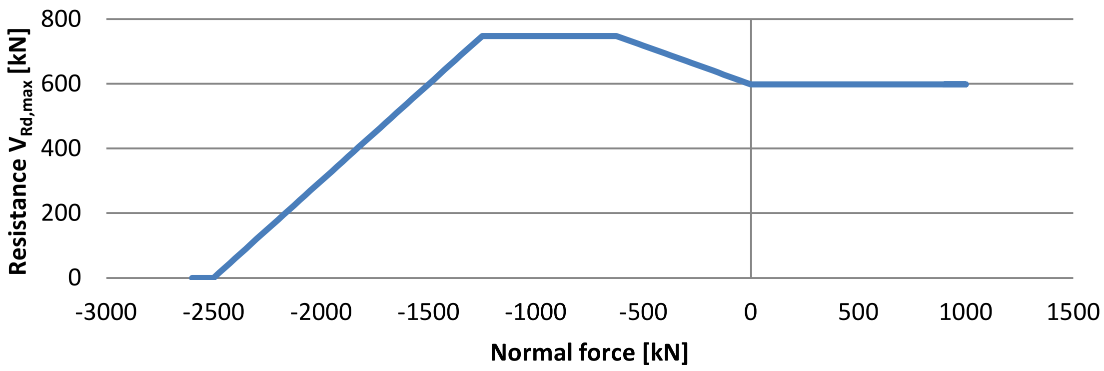


Fig. 1.20 – Dependency curve of shear resistance $V_{Rd,max}$ to normal force

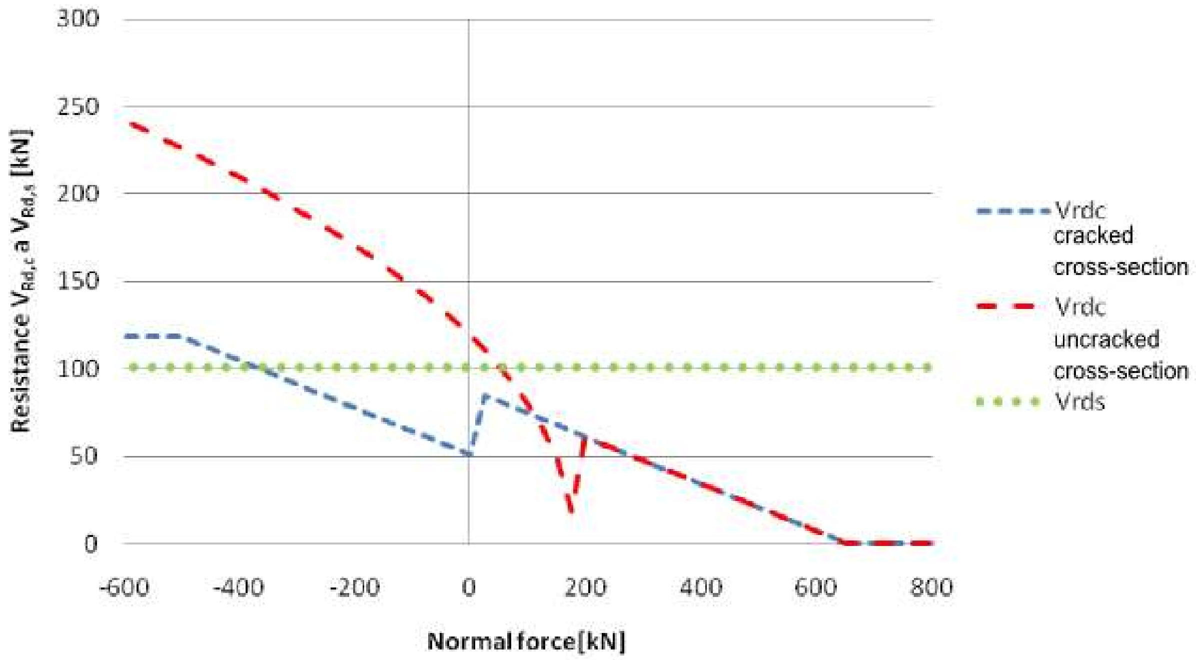


Fig. 1.21 – Dependency of shear resistances $V_{Rd,c}$ a $V_{Rd,s}$ to normal forces

1.3. Torsion

1.3.1. Calculation assumptions

The behaviour of reinforced concrete section subjected to torsion can be divided into two categories - before and after the time when the cracks may first be expected to occur

Before a crack the cross-section behaves about as an elastic material. Torsion stress can be expressed by formula

$\tau = \frac{T_{Ed}}{W_t}$ where W_t je sectional module in torsion.

Cracks in the unreinforced member due to principal tensile torsion stress are also ultimate limit state. The behaviour of reinforced concrete section subjected to torsion can be described on the basis of a thin-walled closed section, see Fig. 1.22. Determination of thin-walled cross-section dimensions is described in caption 1.3.

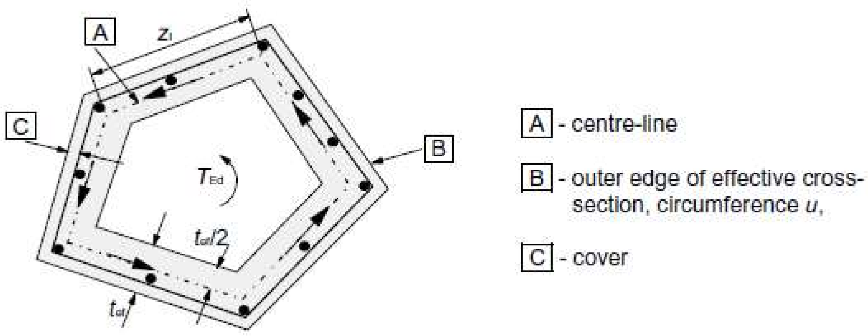


Fig. 1.22 – Equivalent thin-walled cross-section

1.3.2. Calculation procedure

The process of reinforced concrete check for torsion is very similar to the check for shear. First of all, we check the concrete resistance. If the concrete check is satisfying, the reinforcement can be designed using the detailing rules. Otherwise, we need to verify the reinforcement and compressive diagonal resistance by calculation.

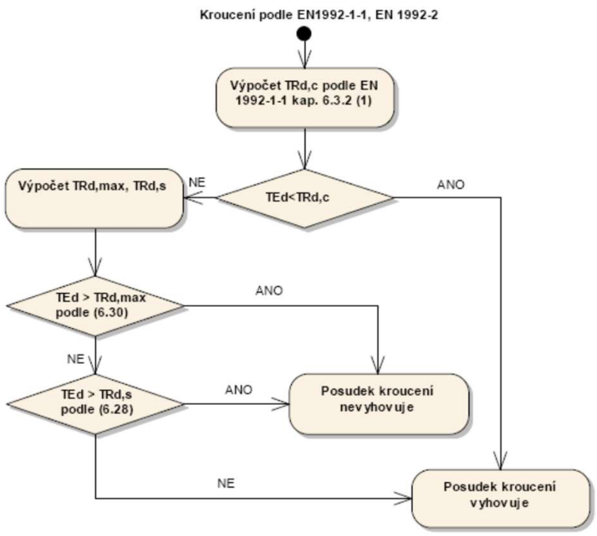


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Fig. 1.23 - Process diagram for torsion check

1.3.3. Resistance

Shear flow in wall of thin-walled cross-section under torsion can be expressed as:

$$\tau_t t_{ef} = \frac{T_{Ed}}{2A_k},$$

Shear force in wall of thin-walled cross-section can be expressed as:

$$V = \tau_t t_{ef} z ,$$

Where τ Shear flow in wall,

t_{ef} is the effective wall thickness,

z is the side length of wall,

T_{Ed} is the torsion moment,

A_k is the area enclosed by the centre-lines of the connecting walls, including inner hollow areas.

Torsion cracking moment, which may be determined by setting f_{ctd} to previous expression. Thus we get expression for the resistance in torsion without torsion reinforcement.

$$T_{Rd,c} = 2A_k t_{ef} f_{ctd} \ ,$$

where f_{ctd} design axial tensile strength of concrete

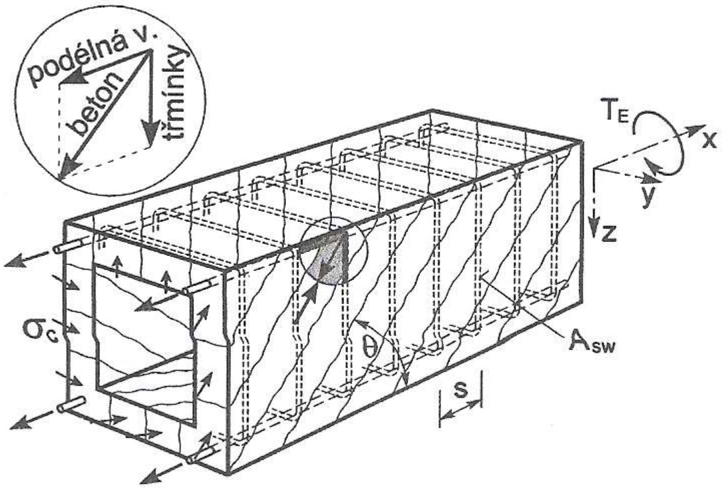


Fig. 1.24 – Principles of Truss analogy for member under torsion moment

The member resistance with torsion reinforcement is composed from the resistance of compressive concrete diagonals which is based again on truss analogy method. Compressive stress in diagonal can be expressed with help of shear force in wall of thin-walled cross-section on wall surface which is in consideration, i.e.

$$\sigma_c = \frac{\frac{T_{Ed} z}{2A_k \sin\theta}}{z\, t_{ef} \cos\theta} = \frac{T_{Ed}}{2A_k t_{ef} \sin\theta \cos\theta} \ .$$

Substitution of $\sigma_c = \alpha_{cw} \nu f_{cd}$ and $T_{Ed} = T_{Rd,max}$ and expressing of $T_{Rd,max}$ we get equation for compressive diagonal resistance

$$T_{Rd,max} = 2\ \nu\ \alpha_{cw}\ f_{cd}\ A_k\ t_{ef}\ \sin\theta\ \cos\theta \ ,$$

where ν = 0,6 pro $f_{ck} \leq$ 60MPa or $0{,}9 - f_{ck}/200$ for $f_{ck} >$ 60MPa,
α_{cw} coefficient which takes account the state of compressive stress in compression chord,
f_{cd} design value of concrete compressive strength

the shear reinforcement resistance subjected to torsion is again based on stress in compression diagonal. The stirrup force is equal to stress in compressed diagonal on the area which corresponds to the particular stirrup line, i.e.

$$A_{sw} f_{ywd} = \frac{T_{Ed}}{2A_k t_{ef} \sin\theta \cos\theta}\ t_{ef}\ s \sin^2\theta = \frac{T_{Ed}\ s}{2A_k \cot\theta}$$

Substituting $T_{Ed} = T_{Rd,s}$ and expressing $T_{Rd,s}$ we get equation

$$T_{Rd,s} = 2A_k \frac{A_{sw} f_{ywd}}{s} \cot\theta \ .$$

If the longitudinal and shear reinforcement quantity is known, we can define angle θ by expression

$$\tan^2\theta = \frac{\frac{A_{sw} f_{ywd}}{s}}{\frac{A_{sl} f_{yd}}{u_k}}$$

Substitution for $T_{Rd,s}$ we get

$$T_{Rd,s} = 2A_k \sqrt{\frac{A_{sw}}{s} f_{ywd} \frac{A_{sl}}{u_k} f_{yd}}\ ,$$

Where

A_{sw}	shear reinforcement area,
s	is the radial spacing of stirrups of shear reinforcement,
f_{ywd}	is the effective design strength of the shear reinforcement,
A_{sl}	longitudinal reinforcement area
u_k	is the outer circumference of the cross-section,
f_{ywd}	is the effective design strength of the longitudinal reinforcement.

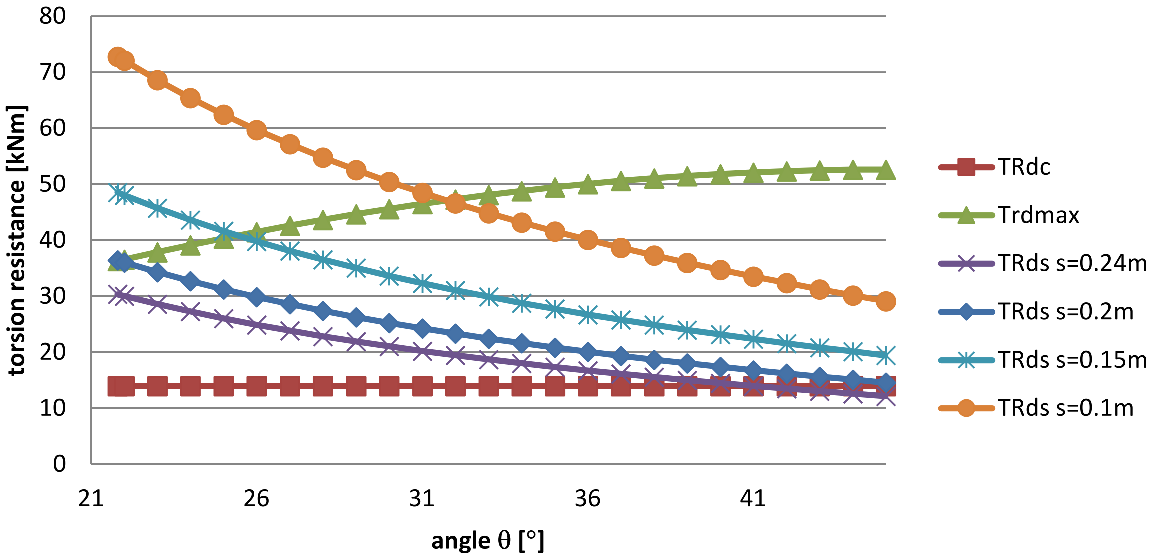
The force in longitudinal reinforcement can be deducted from The shear force in a wall of a section subject to a pure torsional moment, which is give as

$$V = \frac{T_{Ed}}{2A_k} u_k .$$

That force is transformed to longitudinal direction and we get

$$F_l = \frac{T_{Ed} u_k}{2A_k \tan}.$$

The permitted range of the values for angle θ is similar to shear check, i.e. $1 \leq \cot\theta \leq 2{,}5$. Dependency between resistances can be seen on Figobr. 1.25. The picture shows that with increasing the angle θ is growing the resistance $T_{Rd,max}$, is decreasing resistance $T_{Rd.s}$ and resistance $T_{Rd,c}$ is constant, since it is not based on the truss analogy method.



obr. 1.25 - Závislost únosnosti průřezu v kroucení na úhlu θ

1.3.4. Calculation of cross-section characteristics for torsion

To check the cross-section for torsion is necessary to establish so-called equivalent thin-walled closed section. In determining the dimensions of the equivalent thin-walled cross-section is assuming a rectangular shape. For the true area of rectangle states A = b·h and for the circumference of rectangle u = 2 (b +h).
Using these two equations can provide alternative thin rectangle-shaped area and circumference of the original cross-section. Solving two equations with two unknowns get

$b = \frac{-u \pm \sqrt{u^2 - 16A}}{-4}$ a $h = \frac{(u - 2\mathrm{b})}{2}$.

The wall thickness of the effective cross-section can be define from circumference and section area as

$t = \mathrm{A/u}$

Then the area and circumference defined by centre line of the effective cross-section:

$A_k = (\mathrm{h} - \mathrm{t})\,(\mathrm{b} - \mathrm{t})$

$u_k = 2\big((\mathrm{h} - \mathrm{t}) + (\mathrm{b} - \mathrm{t})\big).$

The problem with this method is for cross-section of type T with a wide plate when the overall area and circumference is taken to calculate the dimensions (including this plate). In the future versions of IDEA RCS program, the selection of the most massive cross-section part will be enabled, which will be used to check the torsion.

1.1. Interaction

The term "interaction" in this case means the interaction of shear, torsion, bending and normal force.

1.1.1. Interaction shear and torsion

The resistance of member subjected to shear and torsion interaction is composed of several parts. The resistance of concrete can be expressed as (6.31) in Article 6.3.2 (5) [2]

$$\frac{V_{Ed}}{V_{Rd,c}} + \frac{T_{Ed}}{T_{Rd,c}} \leq 1{,}0.$$

If the above condition is satisfied then only minimum shear reinforcement is required according to rules in article 9.2.1.1) Shear and torsion interaction will be carried out by concrete. If the above condition is not satisfied, the shear and longitudinal reinforcement must be verified.

Longitudinal reinforcement force from shear and torsion can be expressed by

$$F_{stl} = \frac{V_{Ed}}{\tan\theta} + \frac{T_{Ed}\, u_k}{2\, A_k \tan\theta},$$

Particular force component are derived in 1.2.3 and **Chyba! Nenalezen zdroj odkazů.**. The force must be less than

$$F_{stl,max} = \Sigma A_{sl} f_{yd}.$$

Force in shear reinforcement from shear and torsion can be expressed as

$$F_{stw} = \left(\frac{V_{Ed}}{n_c\, z} + \frac{T_{Ed}}{2\, A_k}\right) \tan\theta,$$

Which must be less than

$$F_{stw,max} = A_{sw} f_{ywd}.$$

The last condition for the resistance of the concrete struts is given by expression (6.29) according to art. 6.3.2 (4) v [2]

$$\frac{V_{Ed}}{V_{Rd,max}} + \frac{T_{Ed}}{T_{Rd,max}} \leq 1{,}0.$$

where:

T_{Ed} is the design torsional moment
V_{Ed} is the design transverse force
$T_{Rd,max}$ is the design torsional resistance moment
$V_{Rd,max}$ is the maximum design shear resistance

1.1.2. Interaction shear, torsion and bending

The resistance of member subjected to shear, torsion and bending effects is also based on truss analogy method. The reinforcement designed on bending must carry out also shear and torsion. As stated above, the longidutinal reinforcement force from shear and torsion is equal to:

$$F_{stl} = \frac{V_{Ed}}{\tan\theta} + \frac{T_{Ed}\, u_k}{2\, A_k \tan\theta}.$$

The algorithm used in the RSC program will transfer that force to the strain of reinforcement. That strain we will add to strain from bending and verify ultimate limit state checks.
Further studies were done to compare methods which taking into account the effect of shear on the longitudinal reinforcement.

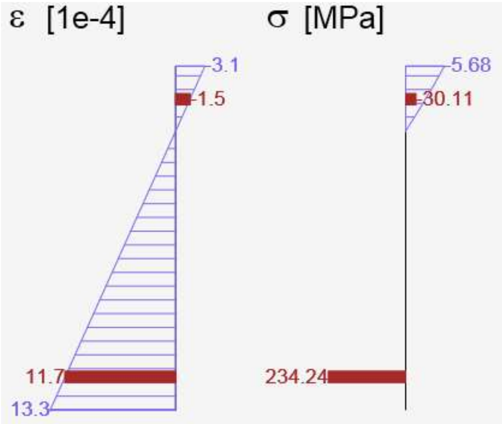


Fig. 1.26 – Without shear influence

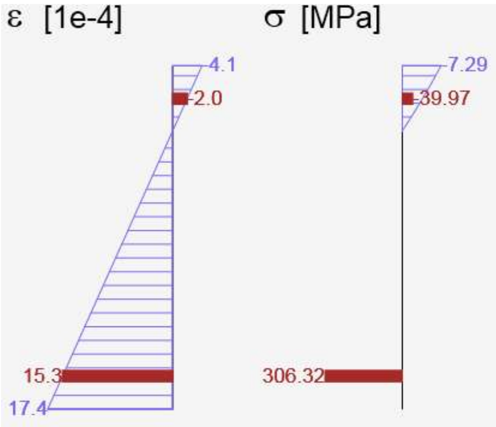


Fig. 1.27 - Shifting the moment curve according to 9.2.1.3 [2]

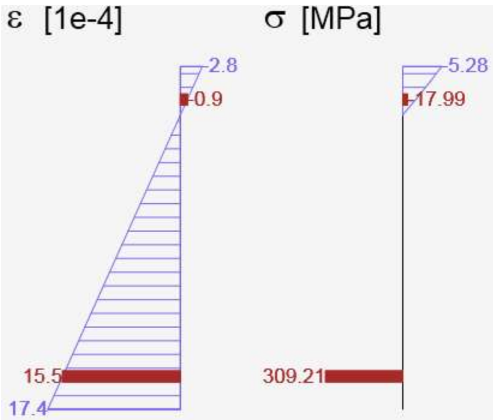


Fig. 1.28 - Additional tensile force from shear is added as load effect

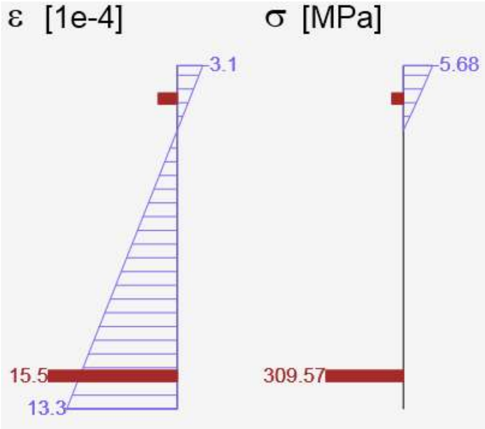


Fig. 1.29 - Additional tensile force from shear is added as strain into reinforcement (used in RCS program)

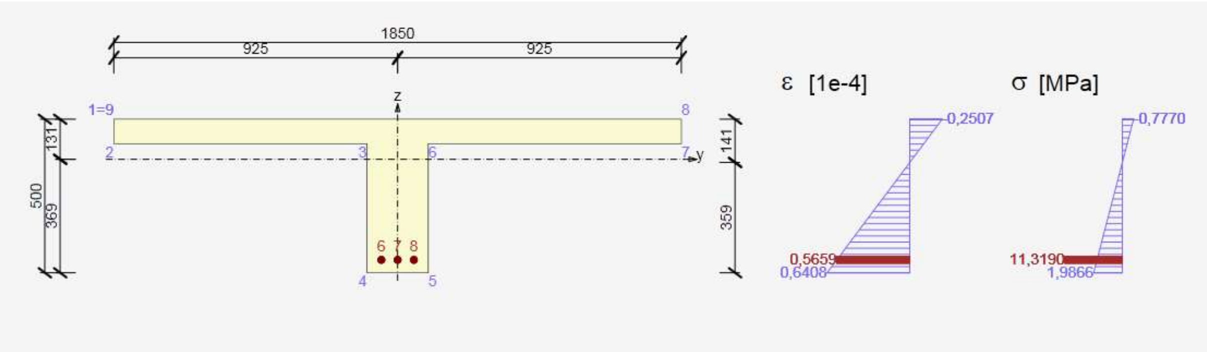
2. Serviceability limit state (SLS)

2.1. Calculation assumptions for Serviceability limit state

The following assumptions are applied in the calculations according to caption 7.2 Stress limitation, 7.3.4 Crack width calculation, 7.4 Deflection control EN [2]

Within the calculation the serviceability limit state we deals with two states that differ only in the tensile strength of concrete

1. Uncracked cross-section
    a. the tensile strength of the concrete is not ignored.
    b. Concrete stress is directly proportional to the distance to neutral axis (linear stress distribution).
    c. Reinforcement stress is directly proportional to the distance to neutral axis (linear stress distribution).
    d. Concrete tensile stress is limited by value $f_{ct,eff}$ according to art. 7.1 (2) [2].



obr. 2.1 – Uncracked section

2. Fully cracked cross section
    a. the tensile strength of the concrete is ignored.
    b. Concrete stress is directly proportional to the distance to neutral axis (linear stress distribution).
    c. Reinforcement stress is directly proportional to the distance to neutral axis (linear stress distribution).

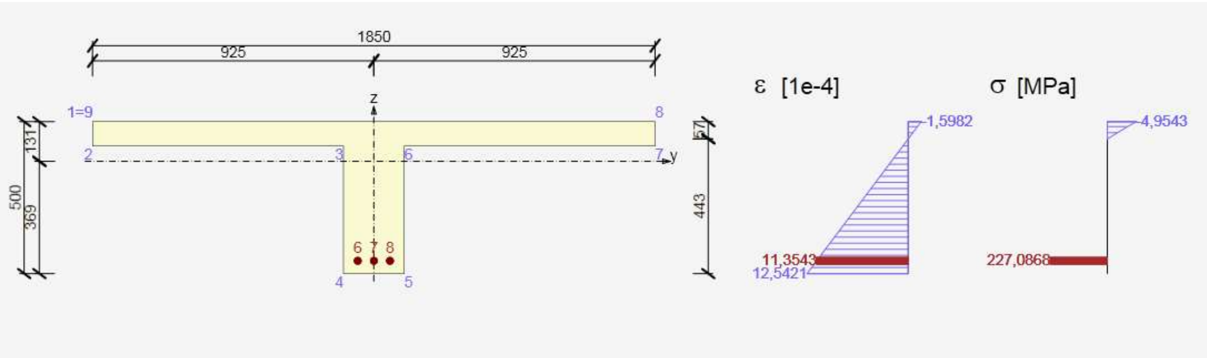


Fig. 2.2 - Cracked cross-section

2.2. Stress limitation check

Stress limitation check introduces Eurocode as one of the new checks in civil engineering. (At old Czech national code CSN 736 207 it is called as check of allowable stress). The check is based on general assumptions acc. to cap. 2.1, where two states of cross-section are resolved. Uncracked section (the tensile strength of the concrete is not ignored) and fully cracked section (the tensile strength of the concrete is ignored). The solution with ignored concrete tensile strength is considered under the assumptions of article 7.1 (2) EN. It should be noted that the checks differs from the check of allowable stress CSN 736207 mentioned above.

When calculating the stress and deflections it is considered uncracked section, if the tensile stress in bending does not exceed $f_{ct,\,eff}$. The value of $f_{ct,\,eff}$ can be considered as f_{ctm} or $f_{ctm,fl}$ provided that in calculating the minimum tensile reinforcement was used the same value. When calculating the crack width and and tensile strengthening it is used f_{ctm} value.

As part of this check we deals with four basic cases in terms of stress limit

1) 7.2 (2) Compressive stress in members exposed to environments of exposure classes XD, XF and XS should be limited:

$|\sigma_c| \leq k_1 f_{ck}$ *k_1=0.6,*

2) 7.2 (3) The stress in the concrete under the quasi-permanent loads is limited:

$|\sigma_c| \leq k_2 f_{ck}$ *k_2=0.45,*

3) 7.2 (5) Tensile stresses in the reinforcement under the characteristic combination of loads shall be limited:

$|\sigma_s| \leq k_3 f_{yk}$ *k_3=0.6,*

7.2 (5) Where the stress is caused by an imposed deformation, the tensile stress should not exceed:

$|\sigma_s| \leq k_4 f_{yk}$ *k_4=1,*

Where

values k_1, k_2, k_3, k_4 for use in a Country may be found in its National Annex. The recommended values are 0,8; 1 and 0,75 respectively,

f_{yk} characteristic yield stress of the reinforcement,

f_{ck} characteristic cylinder strength fck determined at 28 days.

2.3. Crack control

Within the checks of structure in terms of cracks control according to [2] may be possible to do a few calculations.

- Minimum reinforcement are as 7.3.2.

- Control of cracking without direct calculation 7.3.3
  - Maximum bar spacing,
  - Maximum bar diameters.
- Calculation of crack widths 7.3.4.

Calculation of crack widths is the most precise from all mentioned above. Calculations of Minimum reinforcement areas 7.3.2 and Control of cracking without direct calculation 7.3.3 are based on Calculation of crack widths 7.3.4. For this reason, we will further follow only the crack width calculation according to7.3.4.

2.3.1. Crack width w_k calculation

Basic assumptions to calculate the crack width are mentioned in 2.1.

The crack width, w_k may be calculated from Expression:

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$$

Where according to definition in [2]

$s_{r,max}$ is the maximum crack spacing;

ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening. Only the additional tensile strain beyond the state of zero strain of the concrete at the same level is considered,

ε_{cm} is the mean strain in the concrete between cracks.

Following deduction will give us the basic:

$$\varepsilon_{sm} = \varepsilon_{s2} - \Delta\varepsilon_s = \varepsilon_{s2} - \beta\Delta\varepsilon_{sr}$$

ε_{sm} calculates IDEA RCS program as a difference between strain of reinforcement in the crack and strain of reinforcement, which express the influence of concrete between cracks,

ε_{cm} is calculated as strain in concrete providing that concrete tensile strength is neglected.

$$w_m = \varepsilon_m s_{r,m}$$

$$w_m = (\varepsilon_{sm} - \varepsilon_{cm})s_{r,m}$$

Average crack width

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) = s_{r,max}(\varepsilon_{s2} - k_t\Delta\varepsilon_{sr} - k_t\varepsilon_{sr1}) = s_{r,max}\big(\varepsilon_{s2} - k_t(\varepsilon_{sr1} + \Delta\varepsilon_{sr})\big)$$

$$w_k = s_{r,max}(\varepsilon_{s2} - k_t\varepsilon_{sr2}),$$

where

ε_{s2} is the strain in the reinforcement under the relevant combination of loads providing that tensile concrete strength is neglected,

ε_{sr2} is the strain in the reinforcement under the relevant combination of loads at the moment of first appearance of cracks providing that tensile concrete strength is neglected.

$$N_r = f_{ctm(t)}(1+\alpha_e\rho) \Rightarrow \sigma_r = \frac{N_r}{A_s} = \frac{N_r}{\rho} \Rightarrow \varepsilon_{r2} = \frac{N_r}{E_s} = \frac{f_{ctm(t)}}{E_s}\frac{1+\alpha_E\rho}{\rho}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \left(\frac{\sigma_s}{E_s} - \frac{k_t f_{ctm(t)}(1+\alpha_E\rho)}{E_s\rho}\right) \Rightarrow \frac{\sigma_s - k_t\frac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e\rho_{p,eff})}{E_s} \geq 0{,}6\frac{\sigma_s}{E_s}$$

If we modify the formula and use notation in accordance with the standard, we can see that value ε_{sm}- ε_{sm} matches with the formula in the code.

2.3.2. The maximum crack spacing $s_{r,max}$

The next caption will discuss the maximum distance between the cracks $s_{r,max}$. Given that one of the most decisive influences on the crack width is the distance between the reinforcement bars, there are two cases of calculating the maximum distance between the cracks. You can see in the picture

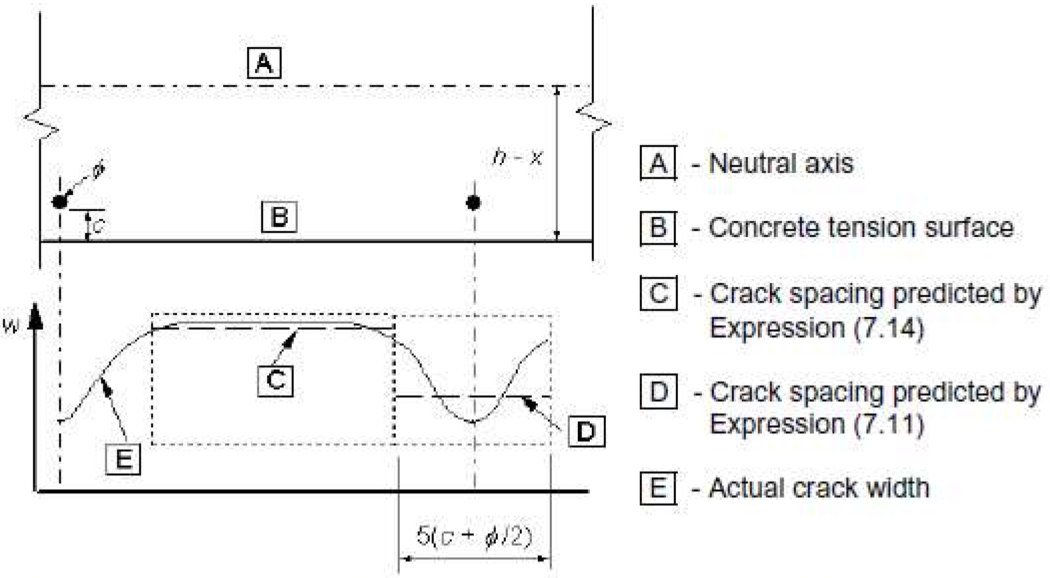


Figure 7.2: Crack width, w, at concrete surface relative to distance from bar

1) In situations where bonded reinforcement is fixed at reasonably close centres within the tension zone (spacing ≤ 5(c+ϕ/2)), the maximum final crack spacing may be calculated from Expression (see Fig. 7.2):

$$s_{r,max} = k_3 c + k_1 k_2 k_3 \phi / \rho_{p,eff} \quad ,$$

Where

ϕ is the bar diameter. Where a mixture of bar diameters is used in a section, an equivalent diameter, ϕ_{eq}, should be used. For a section with $n1$ bars of diameter ϕ_1 and n_2 bars of diameter ϕ_2, the following expression should be used:

$$\phi_{eq} = \frac{n_1\phi_1^2 + n_2\phi_2^2}{n_1\phi_1 + n_2\phi_2}$$

c is the cover to the longitudinal reinforcement;

k_1 is a coefficient which takes account of the bond properties of the bonded reinforcement:
= 0,8 for high bond bars
= 1,6 for bars with an effectively plain surface (e.g. prestressing tendons);

k_2 is a coefficient which takes account of the distribution of strain:
= 0,5 for bending
= 1,0 for pure tension.

For cases of eccentric tension or for local areas, intermediate values of k2 should be used which may be calculated from the relation:
$k_2 = (\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$,

Where ε_1 is the greater and ε_2 is the lesser tensile strain at the boundaries of the section considered, assessed on the basis of a cracked section

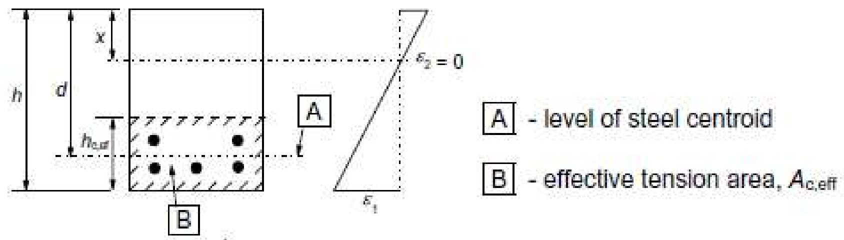
2) Where the spacing of the bonded reinforcement exceeds $5(c+\phi/2)$ (see Figure 7.2 in the code) or where there is no bonded reinforcement within the tension zone, an upper bound to the crack width may be found by assuming a maximum crack spacing

$$s_{r,max} = 1{,}3 - (h - x) \quad .$$

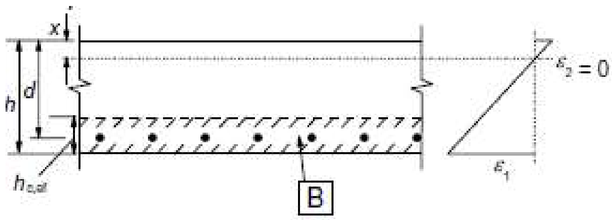
2.3.3. Parameters needed for calculation $\rho_{p,eff}$ used in formulas.

$A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth $h_{c,ef}$, where $h_{c,ef}$ is the lesser of $2{,}5(h - d)$, $(h - x)/3$ or $h/2$ (see Figure 7.1 in EN code);

A_s reinforcement area laying in the area $A_{c,eff}$;

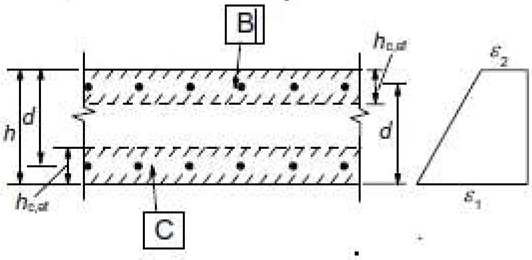


a) Beam



B - effective tension area, $A_{c,eff}$

b) Slab



B - effective tension area for upper surface, $A_{ct,eff}$

C - effective tension area for lower surface, $A_{cb,eff}$

c) Member in tension

2.4. Deflection control

Deflection control can be done two ways

– either by limitation of ratio span/depth according to 7.4.2 [2]
– or by comparing calculated deflection with limit value according to 7.4.3 [2].

2.4.1. Cases where calculations may be omitted

A simple method that can be used for reinforced concrete beams or slabs in civil engineering buildings is based on the control of the span ratio to the effective depth of the cross-section λ to limit ratio of the span to the effective height λ_d, where λ_d is calculated as

$\lambda_d = \kappa_1\ \kappa_2\ \kappa_3 \lambda_{tab}$,

where

κ_1 is 0,8 for flanged sections where the ratio of the flange breadth to the rib breadth exceeds 3,

κ_2 is $7/l_{eff}$ (l_{eff} in meters, see 5.3.2.2 (1) [2]) For beams and slabs, other than flat slabs, with spans exceeding 7 m, which support partitions liable to be damaged by excessive deflections,

κ_3 is $8{,}5/l_{eff}$ (l_{eff} in meters) For flat slabs where the greater span exceeds 8,5 m, and which support partitions liable to be damaged by excessive deflections.

λ_{tab} is calculated using these formulas

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{\frac{3}{2}}\right]\quad if\ \rho \leq \rho_0 \qquad (7.16.\ a)$$

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right]\quad if\ \rho > \rho_0 \qquad (7.16.\ b)$$

where

l/d is the limit span/depth,

K is the factor to take into account the different structural systems,

ρ_0 is the reference reinforcement ratio $\rho_0 = \sqrt{f_{ck}}10^{-3}$,

ρ is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers),

ρ' is the required compression reinforcement ratio at mid-span to resist the moment due to design loads (at support for cantilevers),

f_{ck} is in MPa units.

Expressions (7.16.a) and (7.16.b) have been derived on the assumption that the steel stress, under the appropriate design load at SLS at a cracked section at the mid-span of a beam or slab or at the support of a cantilever, is 310 MPa, (corresponding roughly to fyk = 500 MPa)

Where other stress levels are used, the values obtained using Expression (7.16) should be multiplied by $310/\sigma s$. It will normally be conservative to assume that:

$$310 / \sigma_s = 500 / (f_{yk}\ A_{s,req} / A_{s,prov}) \qquad (7.17)$$

where

σs is the tensile steel stress at mid-span (at support for cantilevers) under the design load at SLS,

$A_{s,prov}$ is the area of steel provided at this section,

$A_{s,req}$ is the area of steel required at this section for ultimate limit state.

Within RCS program is better to calculate σ_s directly, than to do a design of required reinforcement area $A_{s,prov}$. Calculation is much faster, precise and effective in that case.

Values of K for use in a Country may be found in its National Annex Recommended values of K are given in Table 7.4N. Values obtained using Expression (7.16) for common cases (C30, σs = 310 MPa, different structural systems and reinforcement ratios ρ = 0,5 % and ρ = 1,5 %) are also given

Table 7.4N – Basic ratios of span/effective depth for reinforced concrete members without axial Compression

Structural System	K	Concrete highly stressed ρ = 1,5%	Concrete lightly stressed ρ = 0,5%
Simply supported beam, one- or two-way spanning simply supported slab	1,0	14	20
End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	1,3	18	26
Interior span of beam or one-way or two-way spanning slab	1,5	20	30
Slab supported on columns without beams (flat slab) (based on longer span)	1,2	17	24
Cantilever	0,4	6	8

Note 1: The values given have been chosen to be generally conservative and calculation may frequently show that thinner members are possible.
Note 2: For 2-way spanning slabs, the check should be carried out on the basis of the shorter span. For flat slabs the longer span should be taken.
Note 3: The limits given for flat slabs correspond to a less severe limitation than a mid-span deflection of span/250 relative to the columns. Experience has shown this to be satisfactory.

2.4.2. Checking deflections by calculation

Deflection control by calculation can be used for single (statically determinate) structures. We perform direct calculation by substituting the stiffness to analytically derived formulas, which are calculated as stiffness in extremely loaded beam sections.
Further, general methods based on FEM, which can determine the deflection of the general, computational models under general loading. The simplest method is just one step linear calculation with modified stiffness of the finite elements. Stiffness can be determined by the RCS program, see below. Other methods are nonlinear, reflecting not only the nonlinear behaviour of concrete, plasticity as well as second order effects. However, these methods are iterative, time consuming and not always guaranteed convergence. All of the above mentioned methods, however, include the calculation of the stiffness, which the RCS program provides.

2.4.2.1. Calculating the stiffness of uncracked cross-section

Assumptions:

a) According to assumptions 2.1 – uncracked cross-section,
b) It is taken into account the secant value for the modulus of elasticity E_{cm}

Axial stiffness $E\,A_{xI} = A_i\,E_{cm}$,
Bending stiffness $E\,I_{yI} = I_{yi}\,E_{cm}$,
Bending stiffness $E\,I_{zI} = I_{zi}\,E_{cm}$,

where A_i is idealized cross-section area (concrete tensile strength is not neglected),
I_{yi}, I_{zi} moment of inertia related to centre of gravity of idealized cross-section (concrete tensile strength is not neglected)

2.4.2.1. Calculating the stiffness of fully cracked cross-section

Assumptions:

a) According to assumptions 2.1 – cracked cross-section.
b) It is taken into account the secant value for the modulus of elasticity E_{cm}

Axial stiffness $E\,A_{xII} = A_i\,E_{cm}$
Bending stiffness $E\,I_{yII} = I_{yi}\,E_{cm}$
Bending stiffness $E\,I_{zII} = I_{zi}\,E_{cm}$

where A_i is idealized cross-section area (concrete tensile strength is ignored),

I_{yi},I_{zi} moment of inertia related to centre of gravity of idealized cross-section (concrete tensile strength is ignored).

2.4.2.2. Calculating the final stiffnesses

Resulting stiffness corresponds to the intermediate state between the state without cracks 2.4.2.2 and state with fully developed cracks 2.4.2. The elements loaded mainly to flexure the corresponding assumption of behaviour is expressed by relation (7.18) [2]:

$$\alpha = \zeta\alpha_{II} + (1 - \zeta)\,\alpha_I \quad ,$$

where

α is the deformation parameter considered which may be, for example, a strain, a curvature, or a rotation,

α_I, α_{II} are the values of parameter above calculated for state without cracks and with fully developed cracks,

ζ is a distribution coefficient (allowing for tensioning stiffening at a section) given by Expression (7.19):

$$\xi = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2,$$

ζ = 0 for cross-sections without cracks,

β is a coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain:

= 1,0 for a single short-term loading,
= 0,5 for sustained loads or many cycles of repeated loading,

σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section,

σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking.

For loads with a duration causing creep, the total deformation including creep may be calculated by using an effective modulus of elasticity for concrete according to Expression (7.20):

$$E_{c,eff} = \frac{E_{cm}}{1+\varphi(\infty,t_0)} \quad ,$$

where $\varphi(\infty,t_0)$ is the creep coefficient relevant for the load and time interval (see 3.1.4).

Long-term stiffnesses can be calculated from following articles 2.4.2.1, 2.4.2.2 a 2.4.2.3, the only difference is that the secant modulus E_{cm} is replaced by an effective modulus $E_{c,eff}$

Tuhost průřezu pro účinky krátkodobě působícího zatížení

Typ	N [kN]	M y [kNm]	M z [kNm]	EI y [MNm2]	EI z [MNm2]	EA x [MN]
výsledek	0,00	100,00	0,00	25	11	1306
Typ	**Nr [kN]**	**M yr [kNm]**	**M zr [kNm]**	**EI y [MNm2]**	**EI z [MNm2]**	**EA x [MN]**
průřez neporušený trhlinou	0,00	22,31	0,00	96	33	4346
Typ	**N [kN]**	**M y [kNm]**	**M z [kNm]**	**EI y [MNm2]**	**EI z [MNm2]**	**EA x [MN]**
průřez porušený trhlinou	0,00	100,00	0,00	22	9	1147

Mezivýsledky výpočtu tuhosti pro účinky krátkodobě působícího zatížení

A s [mm2]	A st [mm2]	A sc [mm2]	ζ [-]	β [-]	σ sr [MPa]	σ ss [MPa]
1414	785	628	0,95	1,00	71,02	318,37

Průřezové charakteristiky pro účinky krátkodobě působícího zatížení

Typ	A [mm2]	S y [mm3]	S z [mm3]	I y [mm4]	I z [mm4]	t y [mm]	t z [mm]	x [mm]
průřez neporušený trhlinou	160439	-197182	0	3529686915	1229390434	0	-1	106
průřez porušený trhlinou	42345	6082726	0	1670790108	343688693	0	144	106

Mezivýsledky výpočtu tuhosti pro účinky dlouhodobě působícího zatížení

A s [mm2]	A st [mm2]	A sc [mm2]	ζ [-]	β [-]	σ sr [MPa]	σ ss [MPa]
1414	785	628	0,96	0,50	98,04	332,33

2.5. Parametric study made for crack width

Pro následující parametrické studie jsme využili příklad dle Fig. 2.2. Tento příklad je taktéž řešen ve sborníku verifikačních příkladů, kde je rovněž popis zadání příkladu.

2.5.1. First cracking and crack width in relation to load

Within this study, we focused on the analysis of crack width according to the change of internal forces. The study was aimed primarily to verify the results of the IDEA RCS program on extensive set of input values. Study results may also serve for a deeper understanding of the connections in the calculation of crack width according to ČSN EN [2].

The change of internal forces is done in range 5 – 195 kNm, and always in combination with the normal force N = 100kN to -500kN. The curve of crack width is almost linear, and curves are similar. The increase in normal force of constant growth is reflected in the constant increase in crack width.

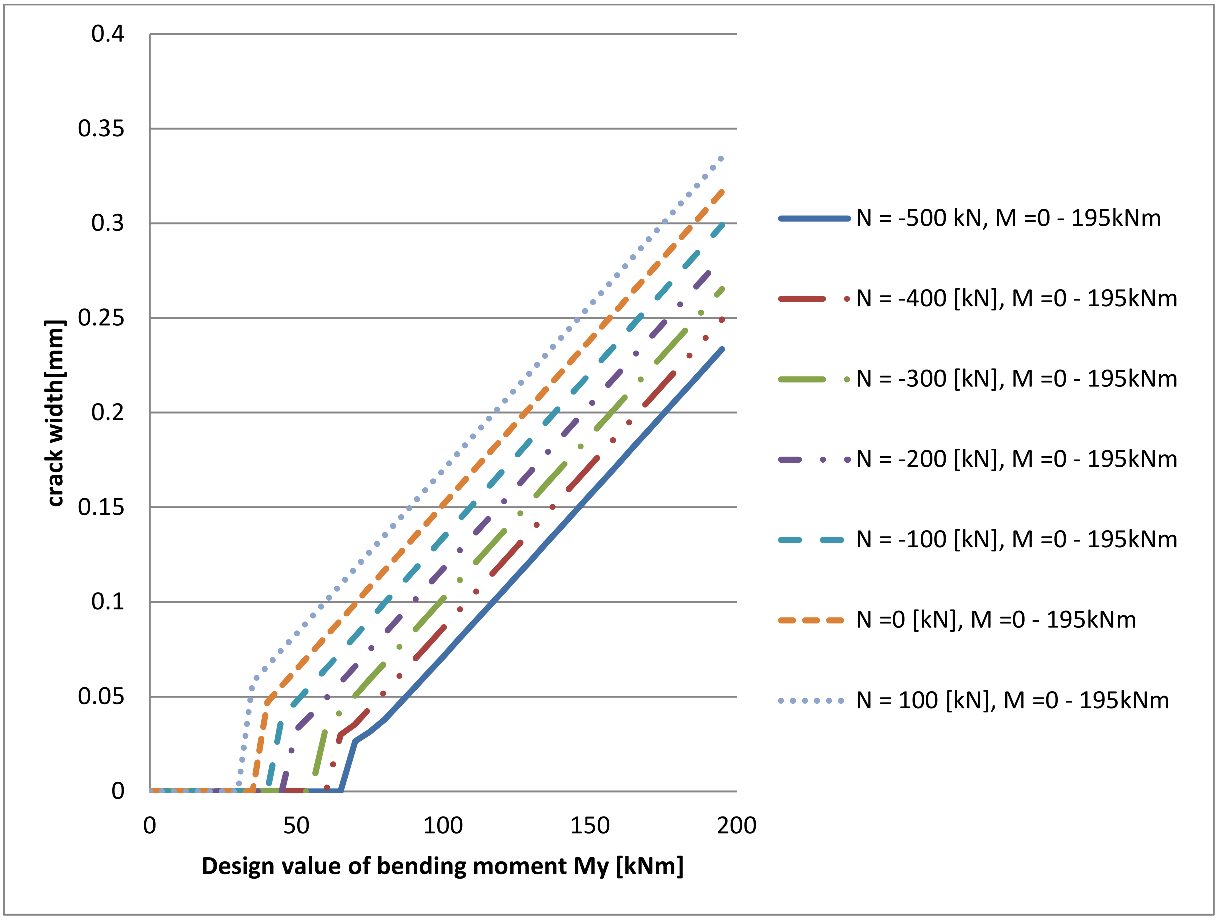


Fig. 2.3 – Parametric study - influence N, M on crack width

2.5.1. First cracking and crack width in relation to tensile reinforcement area

In this study, the bending moments were changed from 0 to 195 kNm. As we can see from the chart, the crack width decreases when increasing the reinforcement area. By increasing the area of reinforcement decreases the stress in the reinforcement, which leads to a reduction in crack width. Increasing the width of the crack is almost linear with an expanding moment.

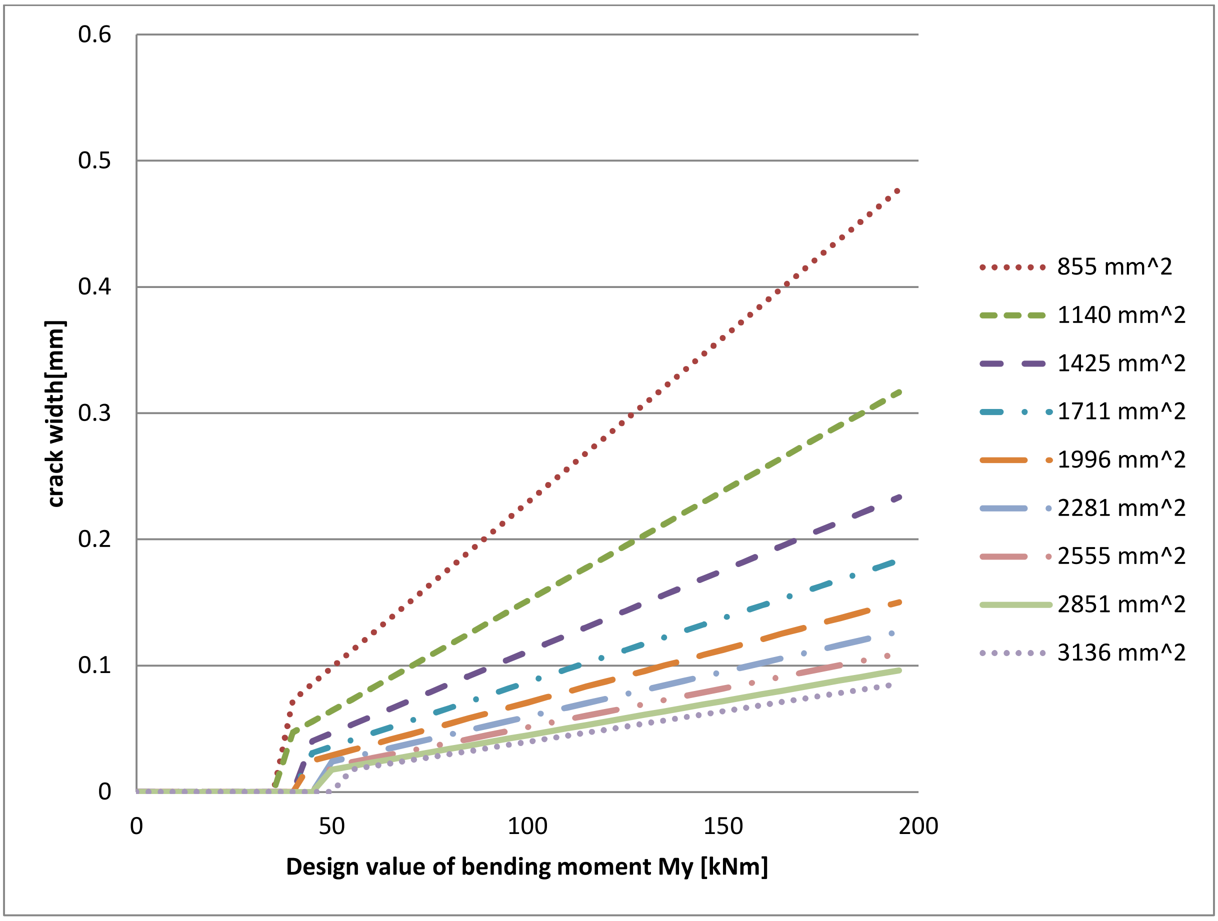


Fig. 2.4 – Parametric study – influence reinforcement area to crack width

2.5.2. First cracking and crack width in relation to bar diameter change

In the last parametric study, we focused only on the change reinforcement bar profile, while the cover and reinforcement area are kept. From the graph you can see a positive impact of the bar diameter decreasing on crack width.

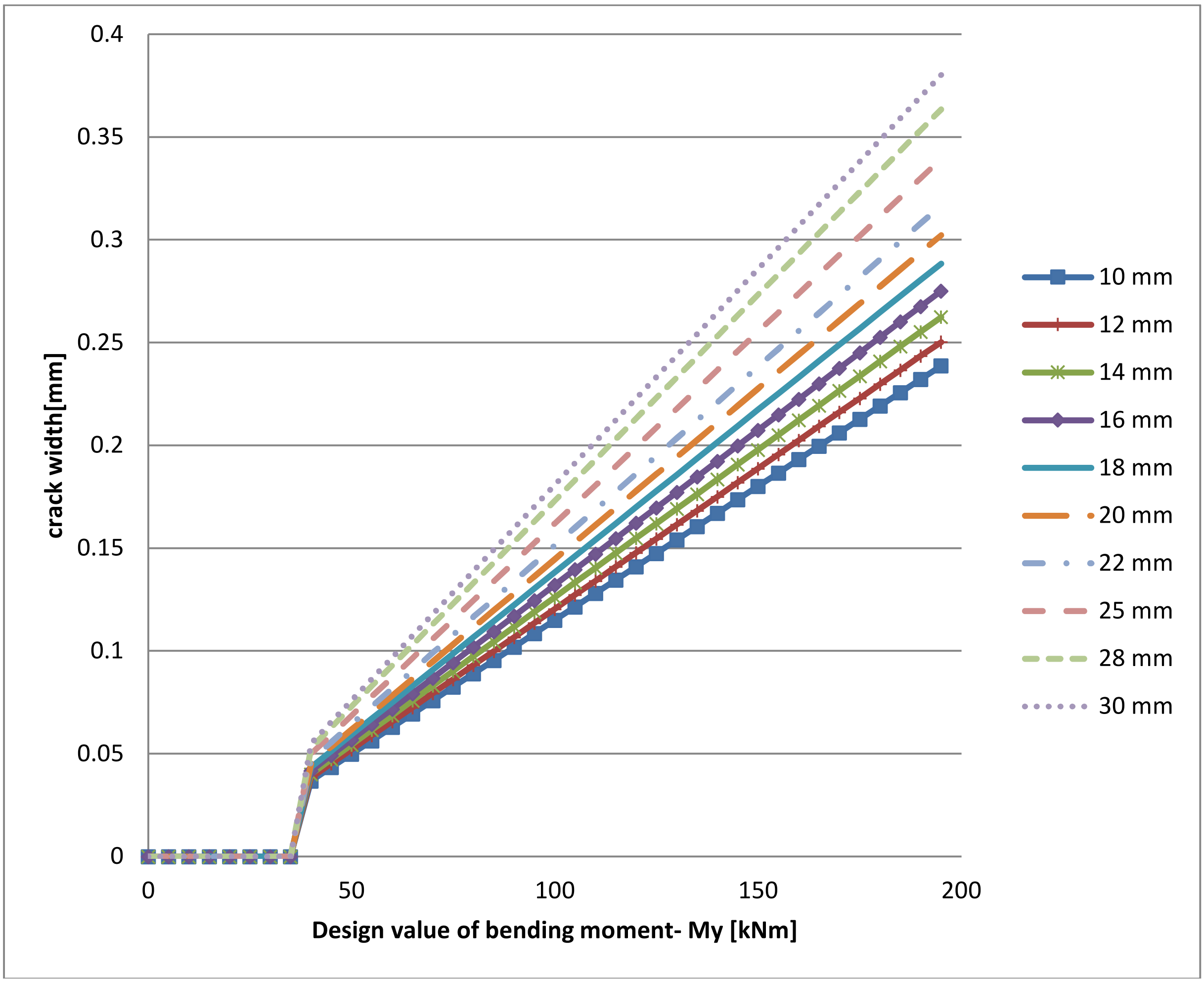


Fig. 2.5 - Parametric study – influence of crack width to bar profile diameter

3. Analysis of compression reinforced concrete members

3.1. General

Uncertainties in geometry and position of loads and second order effects can be taken into account on analyzed structure in meaning of linear elastic analysis of deformed structure or nonlinear analysis of deformed structure or by first order linear analysis with influence of geometric imperfections and second order effects according to EN 1992. In following text, we describe methods that can be applied for the cross-section checks of compression members by IDEA RCS program.

3.2. Effective length

length used to account for the shape of the deflection curve; it can also be defined as buckling length, i.e. the length of a pin-ended column with constant normal force, having the same cross section and buckling load as the actual member.

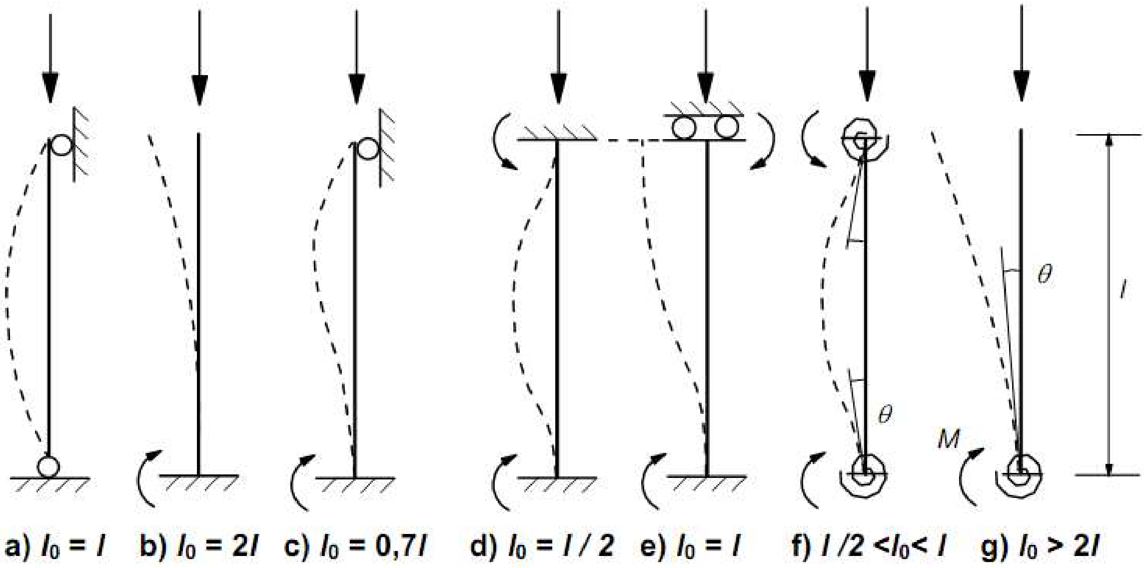


Fig. 3.1 - Examples of different buckling modes and corresponding effective lengths for isolated members (taken from [1])

For compression members in regular frames, the effective length l_0 can be determined in the following way according to article. 5.8.3.2 (3) [1], expressions (5.15) and (5.16):

- Braced members (see Fig. (f), where the translation in restraints at ends 1 and 2 is prevented)

$$l_0 = 0{,}5\, l \cdot \sqrt{\left(1 + \frac{k_1}{0{,}45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0{,}45 + k_2}\right)}.$$

- Unbraced members (see Fig. 3.1 (g))

$$l_0 = l \cdot max\left\{\sqrt{1 + 10 \cdot \frac{k_1 \cdot k_2}{k_1 + k_2}}\ ;\ \left(1 + \frac{k_1}{0{,}45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0{,}45 + k_2}\right)\right\}.$$

k_1, k_2 are the relative flexibilities of rotational restraints at ends 1 and 2 respectively,

$k = (\theta/M) \cdot (EI/l)$,

where θ je is the rotation of restraining members for bending moment M,

EI is the bending stiffness of compression member.

The RCS program makes possible to input effective lengths directly, cases from a) to d) as shown in Fig. 3.1 is possible to calculate.

3.3. Geometric imperfections

The effect of imperfections **must be** taken into account in **ULS** and **need not be** considered for **SLS**.

3.3.1. Calculation procedure

Inclination θ_i according to EN 1992-1-1 art. 5.2 (5), expression (5.1)

$\theta_i = \theta_0 \cdot \alpha_h \cdot \alpha_m$,

where θ_0 is the basic value. The recommended value is 1/200. The minimum value is 1/300,

α_h is the reduction factor for length or height $\alpha_h = 2/\sqrt{l}$ a $2/3 \leq \alpha_h \leq 1$,

α_m is the reduction factor for number of members $\alpha_m = \sqrt{0{,}5(1 + 1/m)}$,

l is the length or height [m], (according to effect which is under consideration),

m is the number of vertical members contributing to the total effect.

Accordingly EN 1992-2, art. 5.2 (105) [2], the inclination is defined as:

$\theta_i = \theta_0 \cdot \alpha_h$,

where θ_0 is the basic value, the recommended value is 1/200

α_h is the reduction factor for length or height $\alpha_h = 2/\sqrt{l}$ a $\alpha_h \leq 1$

l is the length or height [m].

For isolated members, the effect of imperfections may be taken into account in two alternative ways a) or b)

a) As an eccentricity $e_i = \theta_i l_0/2$, where l_0 is the effective length (in this way the effect of imperfections is taken into account in program IDEA RCS)
b) As a transverse force H_i
   - For unbraced members $H_i = \theta_i N$

     Cantilever: $M_i = N\, e_i = N\, \theta_i \frac{l_0}{2} = H_i \frac{l_0}{2}$

   - For braced members $H_i = 2\, \theta_i N$, where N is Normal force

     Hinge supports: $M_i = N\, e_i = N\, \theta_i \frac{l_0}{2} = H_i \frac{l_0}{4}$.

For structures, the effect of the imperfections may be represented by transverse forces,

- $H_i = \theta_i(N_b - N_a)$ Effect on bracing system
- $H_i = \theta_i(N_b + N_a)/2$ Effect on floor diaphragm
- $H_i = \theta_i N_a$ Effect on roof diaphragm

  where *Na* and *Nb* are axial forces.

RCS program is the program for the check of one section, hence it is impossible to distinguish cases of **isolated members** and **structures**. Therefore, the effect of imperfections is considered fundamentally as the eccentricity, which is obtained from the inclination θ_i from vertical$e_i = \frac{\theta_i l_0}{2}$.

The minimum eccentricity under Article 6.1 (4) code [2] for the ultimate limit state can be applied either to the resistance (by reducing the interaction diagram) or the load. This is the case for RCS, where the eccentricity e_0 is taken into account for eccentricity including geometric imperfections, $e_{0Ed} = \max(e_{lin} + e_i;\ e_0)$,

$e_0 = \max(h/30;\ 20\ mm)$, where h is the height of the section.

3.4. Second order effects

Additional action effects caused by structural deformations. When calculating the deflection it should be taken into account the possibility of cracks in concrete and nonlinear material properties, or take these effects into account by reducing stiffness in the method of nominal stiffness.

3.4.1. Neglecting second order effects

Second order effects may be ignored if:

1. they are less than 10 % of the corresponding first order effects (5.8.2 (6) [1]).
2. the slenderness is below a limit slenderness, $\lambda < \lambda_{lim}$ (5.8.3.1 (1) [1]).
3. Vzpěrná únosnost budovy jako celku je větší než celkové svislé zatížení (5.8.3.3 [1]) a kapitola **Chyba! Nenalezen zdroj odkazů.** tohoto textu.

Program RCS controls condition 1 and 2 and if they are fulfilled, the second order effects can be neglected.

3.4.2. Stiffness criterion for isolated members

Limit stiffness, which determines the border between slender and non-slender members is given by expression, art. 5.8.3.1 (1), expression (5.13) [1]

$\lambda_{lim} = 20 \cdot A \cdot B \cdot C/\sqrt{n}$,

Where

$A = 1/(1 + 0{,}2\varphi_{ef})$	(if φ_{ef} is not known, A=0,7 may be used),
$B = \sqrt{1 + 2\omega}$	(if ω is not known, B=1,1 may be used),
$C = 1{,}7 - r_m$	(if r_m is not known C=0,7 may be used),
φ_{ef}	effective creep ratio,
$\omega = A_s f_{yd}/(A_c f_{cd})$	mechanical reinforcement ratio,
A_s	is the total area of longitudinal reinforcement,
A_c	concrete section area,

$n = N_{Ed}/(A_c f_{cd})$ relative normal force,

$r_m = M_{01}/M_{02}$ moment ratio,

M_{01}, M_{02} the first order end moments, $|M_{02}| \geq |M_{01}|$.

In the following cases, r_m should be taken as 1,0:

- for braced members in which the first order moments arise only from or predominantly due to imperfections or transverse loading
- for unbraced members in general

3.4.3. Slenderness of isolated members

The slenderness ratio is defined as follows

$$\lambda = \frac{l_0}{i},$$

where l_0 is effective length

i is the radius of gyration of the uncracked concrete section.

The Code states that for the slenderness ratio calculation it should be considered uncracked concrete section. Usually in practice, the reinforcement of cross-section is not available when we check the stiffness. Using an ideal section for calculating the radius of gyration would be possible to avoid the calculation of second order effects in a broader class of problems. Results of the study for the particular case of the column shows that in case of an ideal cross-section slenderness limit is reached on the reinforcement ratio 0.29, in case of the concrete section from the level 0.33, see Figure3.2. The RCS program is considering concrete section on the safe side.

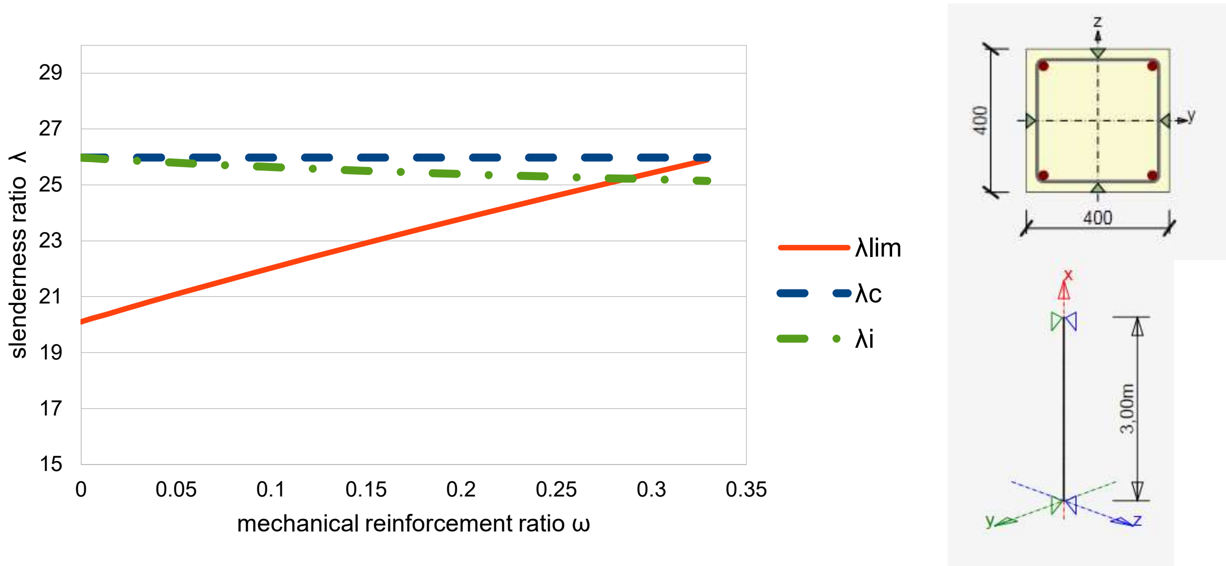


Fig. 3.2 – Comparison of slenderness for concrete and idealized cross-section

3.4.4. Global second order effects in buildings

Global second order effects in buildings may be ignored if the building buckling strength is more than overall vertical load, according to art. 5.8.3.3 [1], expression (5.15):

$$F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1{,}6} \cdot \frac{\sum E_{cd} I_c}{L^2},$$

Where

$F_{VE,d}$ je	is the total vertical load (on braced and bracing members),
n_s	is the number of storeys,
L	is the total height of building above level of moment restraint,
E_{cd}	is the design value of the modulus of elasticity of concrete,
I_c	is the second moment of area (uncracked concrete section) of bracing member(s).

The recommended value is 0,31 of k1 is 0,31.
Expression above is valid only if all the following conditions are met:

- torsional instability is not governing, i.e. structure is reasonably symmetrical
- global shear deformations are negligible (as in a bracing system mainly consisting of shear walls without large openings)
- bracing members are rigidly fixed at the base, i.e. rotations are negligible
- the stiffness of bracing members is reasonably constant along the height
- the total vertical load increases by approximately the same amount per storey

The option to neglect global second order effects in buildings is not available in RCS program.

3.4.5. Creep

The effect of creep maybe taken account in a simplified way by means of an effective creep ratio art. 5.8.4 (2), expression (5.19)

$$\varphi_{ef} = \varphi_{(\infty,t0)} M_{0Eqp} / M_{0Ed},$$

Where

$\varphi_{(\infty,t0)}$	is the final creep coefficient according,
M_{0Eqp}	is the first order bending moment in quasi-permanent load combination (SLS),
M_{0Ed}	is the first order bending moment in design load combination (ULS).

The effect of creep may be ignored, i.e. $\varphi_{ef} = 0$ may be assumed, if the following three conditions are met:

- $\varphi_{(\infty,t0)} \leq 2$,
- $\lambda \leq 75$,

$M_{0E}\ /N_{Ed} \geq h$, kde h is the cross section depth in the corresponding direction.

If the conditions for neglecting second order effects according to 5.8.2 (6) or 5.8.3.3 are only just achieved, it may be too unconservative to neglect both second order effects and creep, unless the mechanical reinforcement ratio (ω, see 5.8.3.1 (1)) is at least 0,25

Therefore, when the RCS program calculates the effective creep ratio equal to zero, while the second order effects are less than 10% of first order effects (i.e. they can be neglected), it is advisable to increase the degree of mechanical reinforcement over the value 0.25.

3.4.6. Methods of analysis

The code introduces these calculation methods for second order effects:

- General method, based on non-linear second order analysis includes geometrical nonlinearity
- Method based on nominal stiffness, see 3.4.7.
- Method based on nominal curvature, see 3.4.8.

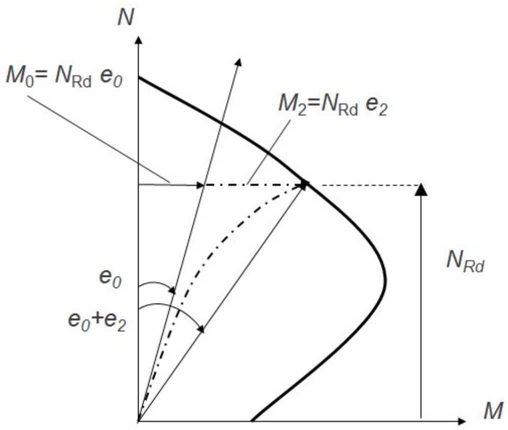


Fig. 3.3 – Second order effects displayed in interaction diagram

3.4.7. Method based on nominal stiffness (5.8.7 [1])

The method defines the stiffness in way that the recalculated first order bending moments can be used for check at the ultimate limit state, i.e. to include the second-order effects. The method considers the influence of geometric nonlinearity but assumed a linear behaviour of the material.

The following model may be used to estimate the nominal stiffness of slender compression members with arbitrary cross section (art. 5.8.7.2 (1) [1]):

$$EI = K_c E_{cd} I_c + K_s E_s I_s,$$

Where

E_{cd} is the design value of the modulus of elasticity of concrete, $E_{cd} = E_{cm}/\gamma_{cE}$, $\gamma_{cE} = 1{,}2$,

I_c is the moment of inertia of concrete cross section,

E_s is the design value of the modulus of elasticity of reinforcement,

I_s is the second moment of area of reinforcement, about the centre of area of the concrete,

K_c is a factor for effects of cracking, creep etc, etc.,

K_s is a factor for contribution of reinforcement.

The following factors K_s a K_c may be used in Expression (5.21), provided $\rho \geq 0{,}002$:

$$K_s = 1,$$

$K_c = k_1 k_2/(1 + \varphi_{ef})$,

Where

$\rho = A_s/A_c$ je	is the geometric reinforcement ratio
A_s	is the total area of reinforcement,
A_c	is the area of concrete section,
φ_{ef}	is the effective creep ratio
$k_1 = \sqrt{f_{ck}/20}$	is a factor which depends on concrete strength class, f_{ck} in MPa,
$k_2 = n \cdot \lambda/170$	is a factor which depends on axial force and slenderness, $k_2 \le 0{,}2$,
$n = N_{Ed}/A_c f_{cd}$	is the relative axial force,
λ	is the slenderness ratio, see 3.4.3.

As a simplified alternative, provided $\rho \ge 0{,}01$, the following factors may be used:

$K_s = 0$,

$K_c = 0{,}3/(1 + 0{,}5\varphi_{ef})$.

The simplified alternative may be suitable as a preliminary step, followed by a more accurate calculation, for example design of reinforcement.

Second order moment

$$M_2 = M_{0Ed} \frac{\beta}{(N_B/N_{Ed}) - 1},$$

kde	M_{0Ed}	is the first order moment with influence of imperfections,
	β	is a factor which depends on distribution of 1st and 2nd order moments,
	N_{Ed}	is the design value of axial load,
	N_B	is the buckling load based on nominal stiffness, $N_B = \frac{\pi^2 EI}{l_0^2}$.

The total design moment, including second order moment, may be expressed as given in art. 5.8.7.3 (1), expression (5.28) [1]

$$M_{Ed} = M_{0E} + M_2 = M_{0Ed}\left[1 + \frac{\beta}{(N_B/N_{Ed}) - 1}\right],$$

For isolated members with constant cross section and axial load, the second order moment may normally be assumed to have a sine-shaped distribution

$\beta = \pi^2/c_0$

Where

c_0 is a coefficient which depends on the distribution of first order moment (for instance, $c_0 = 8$ for a constant first order moment, $c_0 = 9{,}6$ for a parabolic and $c_0 = 12$ for a symmetric triangular distribution etc.).

3.4.8. Method based on nominal curvature (5.8.8 [1])

The method defines the curvature corresponding to deflection under second order effects. This method is primarily suitable for isolated members with constant normal force.

The design moment according to art. 5.8.8.2 [1]

$M_{Ed} = M_{0Ed} + M_2,$

Where

M_{0E}	is the 1st order moment, including the effect of imperfections,
$M_2 = N_{Ed} e_2$	is the nominal 2nd order moment,
N_{Ed}	is the design value of axial force,
$e_2 = \left(\frac{1}{r}\right) l_0^2/c$	is the deflection from second order effects,
$1/r$	is the curvature,
l_0	is the effective length, see 3.2,
c	is a factor depending on the curvature distribution, for constant cross-section, c =10 ($\approx\pi^2$) is normally used.

Curvature for members with constant symmetrical cross sections (incl. reinforcement), the following expression may be used

$1/r = K_r \cdot K_\varphi \cdot 1/r_0,$

Where

K_r	is a correction factor depending on axial load,
K_φ	is a factor for taking account of creep,

$1/r_0 = \varepsilon_{yd}/(0{,}45\, d),$

$\varepsilon_{yd} = f_{yd}/E_s,$

d	is the effective depth, $d = (h/2) + i_s$, If all reinforcement is not concentrated on opposite sides, but part of it is distributed parallel to the plane of bending,
i_s	is the radius of gyration of the total reinforcement area.

Coefficient K_r should be taken as

$K_r = (n_u - n)/(n_u - n_{bal}) \leq 1,$

Where

n	relative axial force, $n = N_{Ed}/(A_c f_{cd})$,
$n_u = 1 + \omega$,	
n_{bal}	is the value of n at maximum moment resistance; the value 0,4 may be used
ω	mechanical reinforcement ratio, $\omega = A_s f_{yd}/(A_c f_{cd})$,
A_s	total reinforcement area,
A_c	concrete section area.

Coefficient K_φ is defined as

$K_\varphi = 1 + \beta\varphi_{ef} \geq 1,$

Where

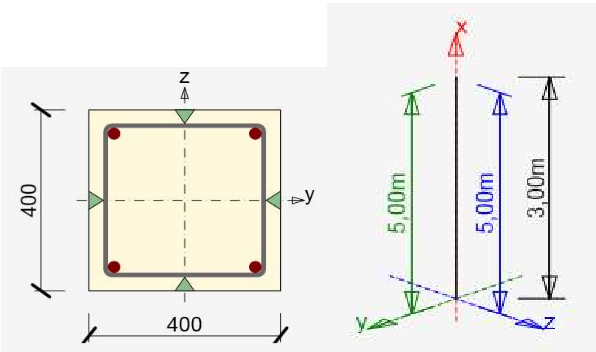
φ_{ef}	effective creep coefficient, see **Chyba! Nenalezen zdroj odkazů.**,

$\beta = 0{,}35 + f_{ck}/200 - \lambda/150$,

λ slenderness ratio, see 3.4.3,

3.4.9. Comparing the nominal stiffness method and nominal curvature method

For the verification and stabilization reasons of above simplified methods is given following comparison.



Bar	Ø [mm]	Material	Y [mm]	Z [mm]
1	25	B 500B	-148	-148
2	25	B 500B	148	-148
3	25	B 500B	148	148
4	25	B 500B	-148	148

Fig. 3.4 – Input of structure for comparing of simplified methods

Figure 3.5, Figure 3.7 and Figure 3.9 shows the dependency of the second-order effects on the axial force. It was assumed a constant linear eccentricity 20, 100 and 200 mm. At the method of nominal stiffness after reaching the critical force the structure will be unstable and the method is terminated. The Figure 3.6, Figure 3.8 and Figure 3.10 shows the resistance of cross-section and the linear eccentricity is drawn and the corresponding second order effects calculated by simplified methods

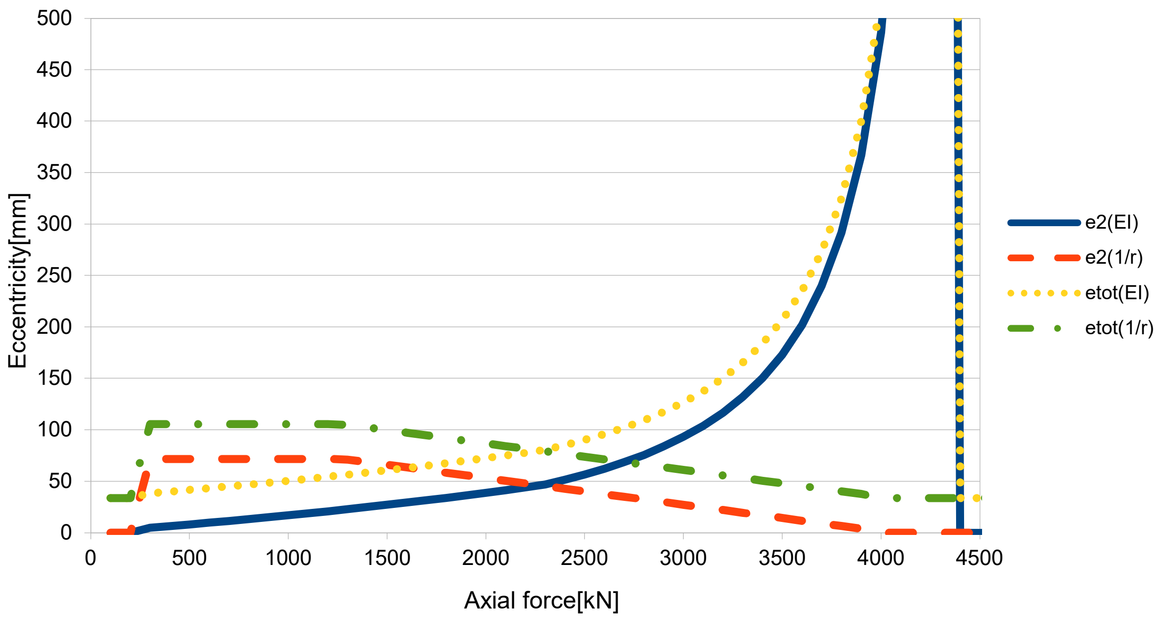


Fig. 3.5 – Influence between Axial forces to second order effects for constant linear eccentricity 20 mm

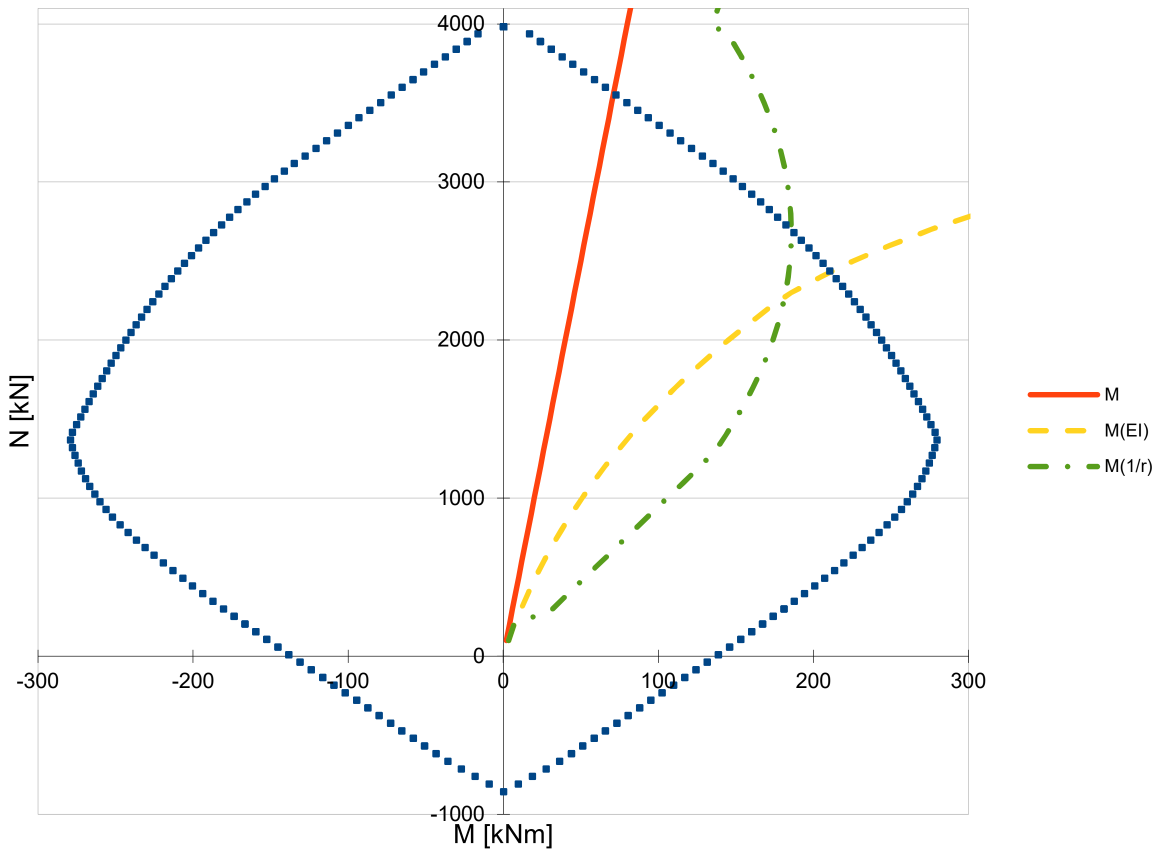


Fig. 3.6 – Design internal forces for constant linear eccentricity 20 mm

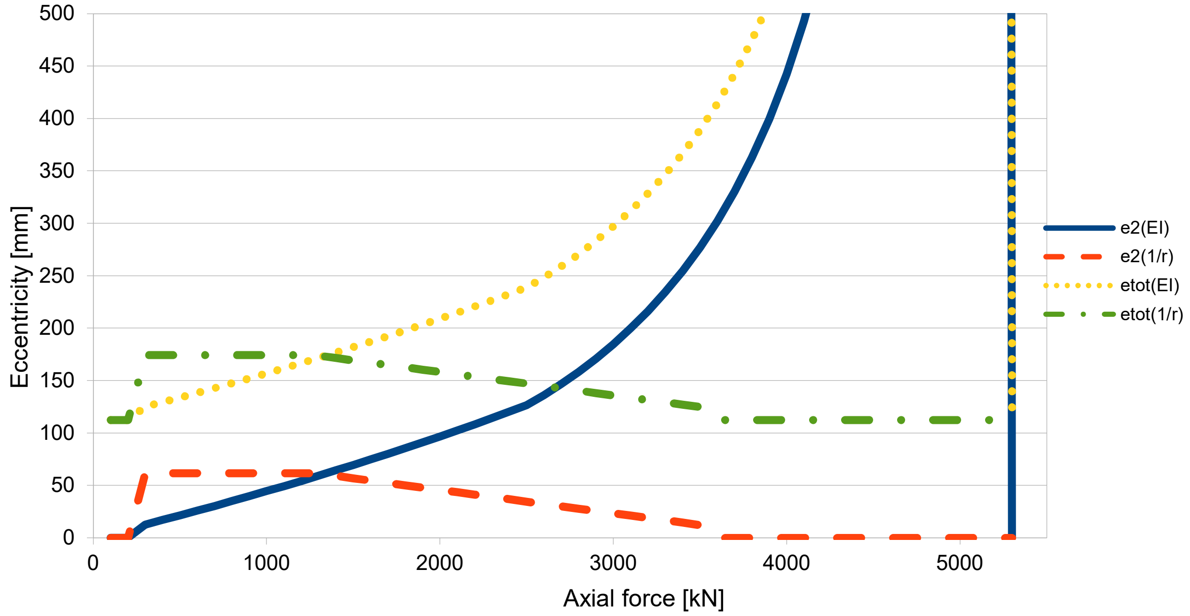


Fig. 3.7 – Influence of axial force to second order effects for constant linear eccentricity 100 mm.

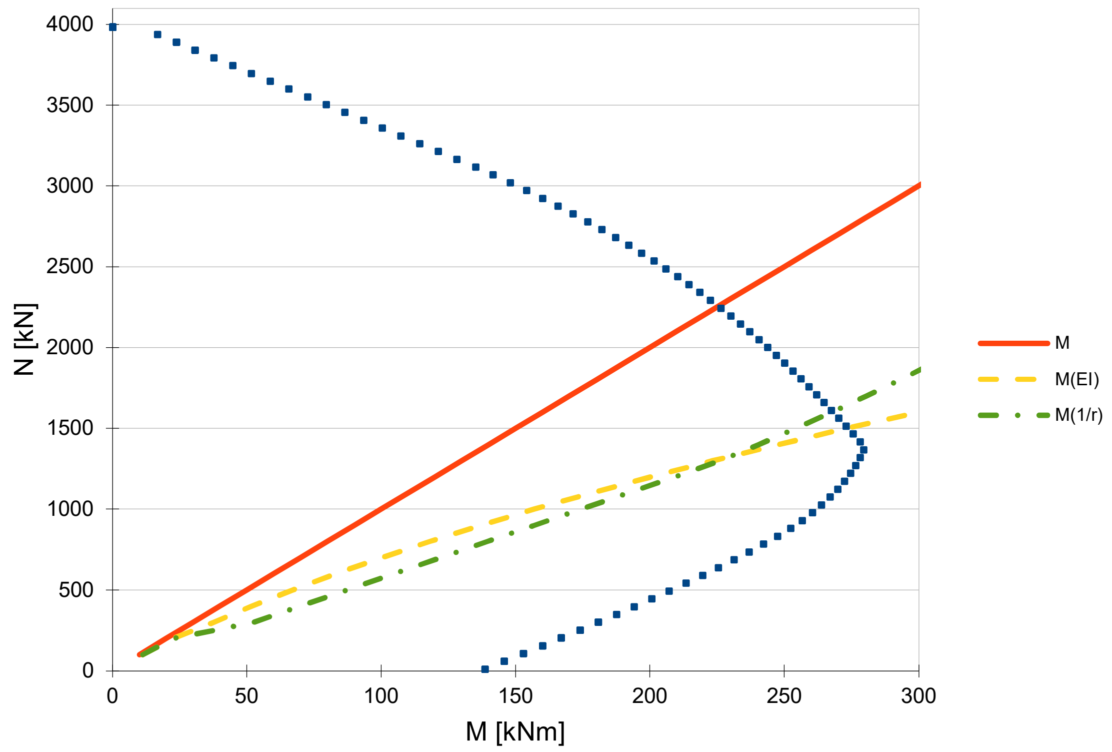


Fig. 3.8 – Design internal forces for constant linear eccentricity 100 mm

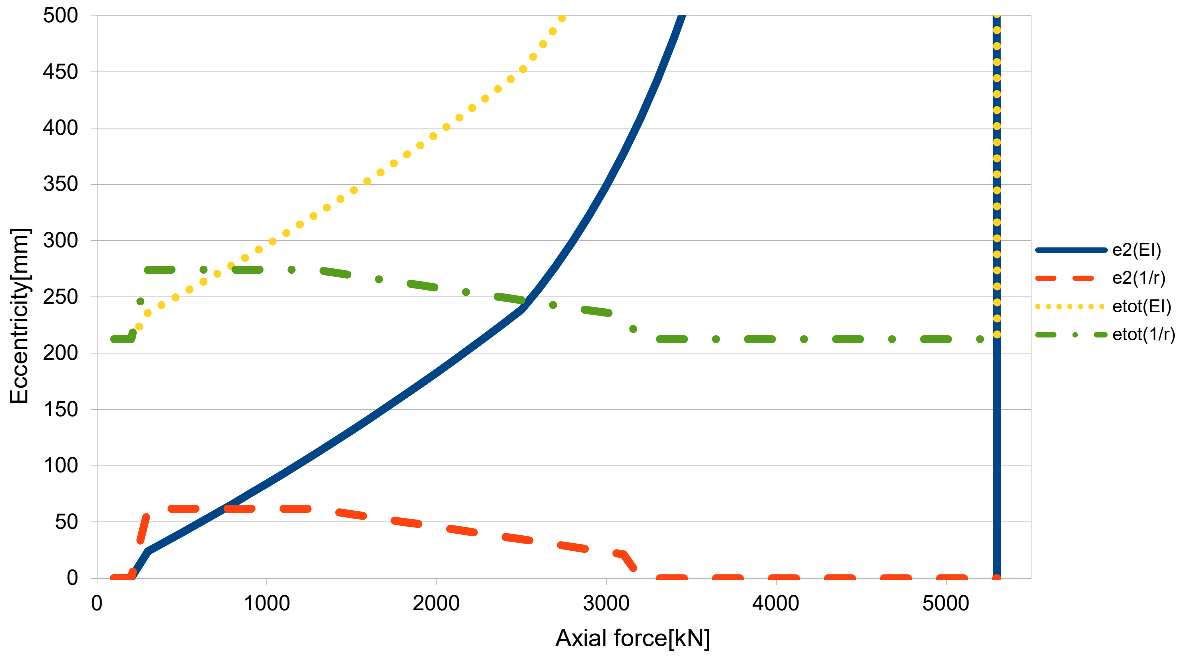


Fig. 3.9 – Influence of axial force to second order effects for constant linear eccentricity 200 mm.

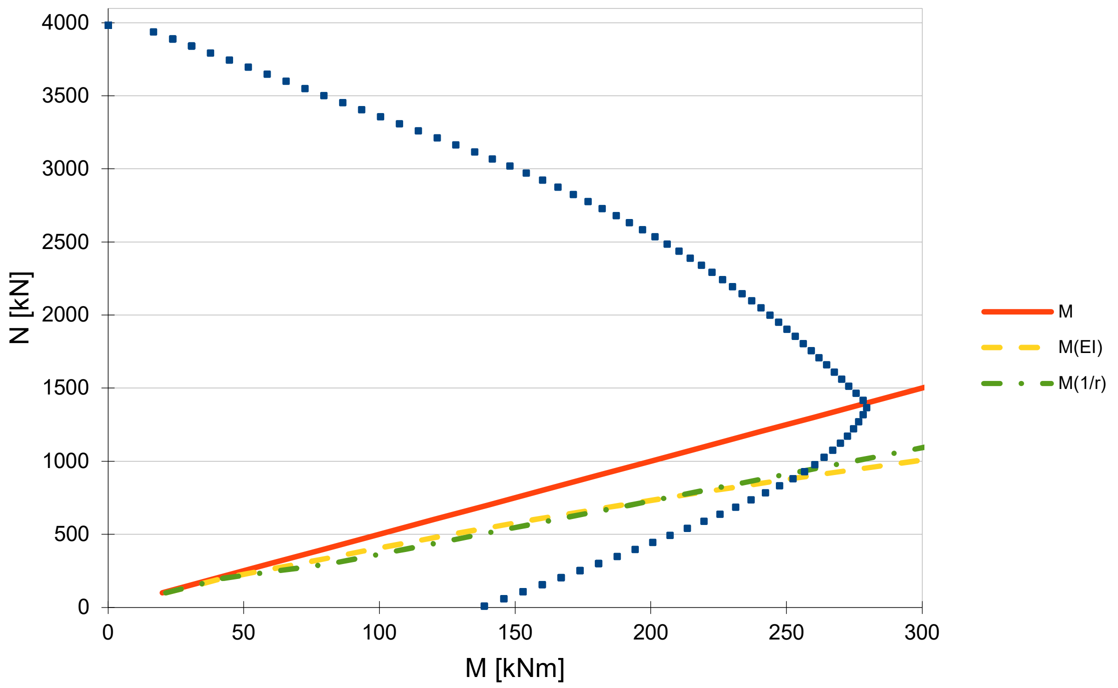


Fig. 3.10 – Design internal forces for constant linear eccentricity 200 mm

On the Fig. 3.11 the influence of second order effects and final eccentricity on reinforcement ratio is shown. Member is loaded by constant axial force 1000 kN and bending moment 50 kNm.

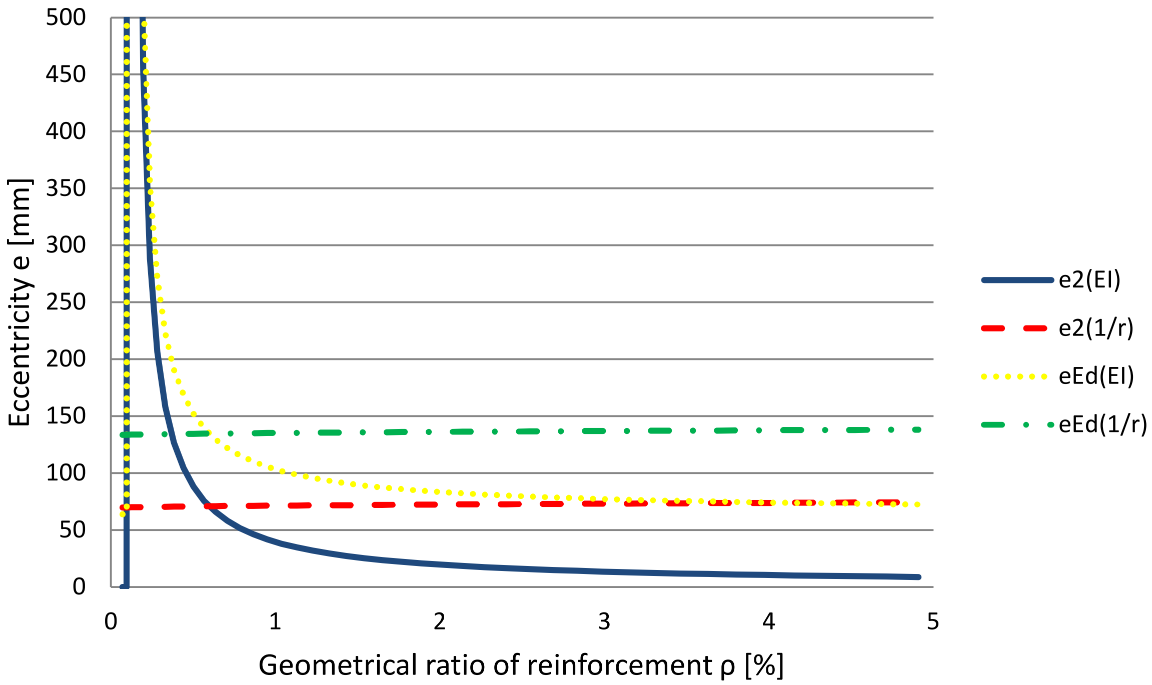


Fig. 3.11 – Dependency of second order effects on reinforcement ratio

Size effect of bending moment on the eccentricity with a constant axial force of 1000kN. Figure 3.12 shows that second order effects quantified by the nominal curvature method depend on the size of the bending moment.

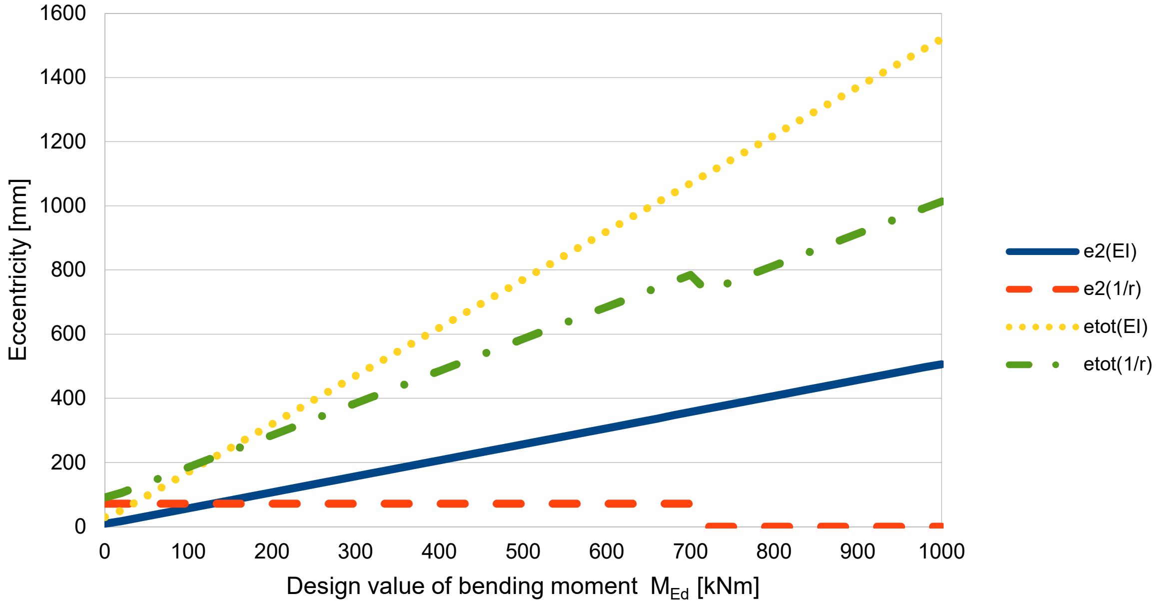


Fig. 3.12 – Dependency of second order effects on reinforcement ratio

Both methods are compared on the following pictures.

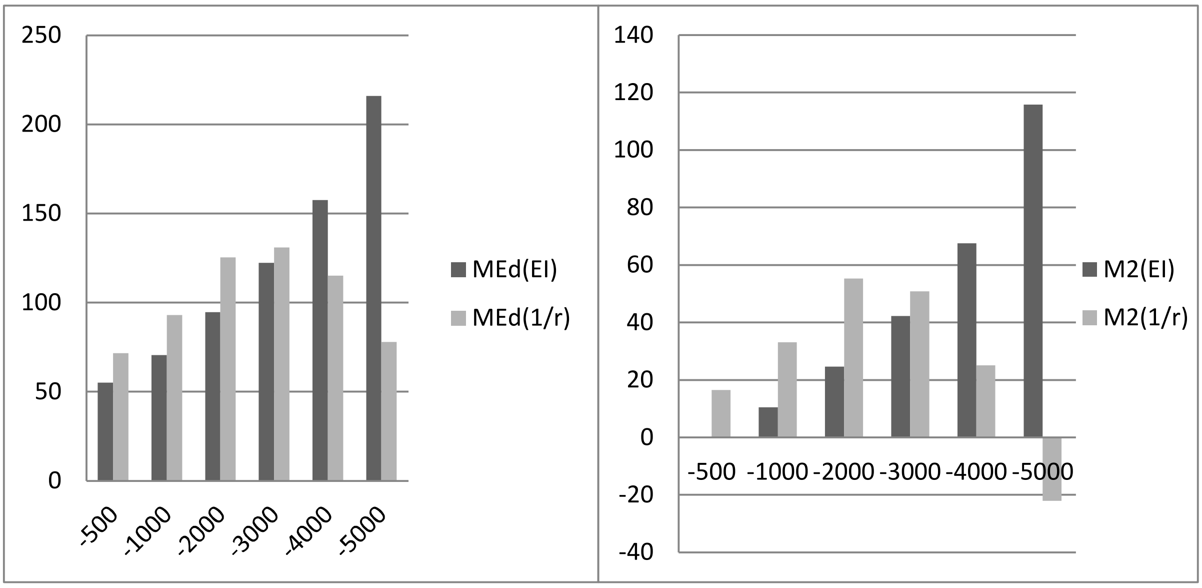


Fig. 3.13 – Simplified method comparison, square column, see Fig. 3.2

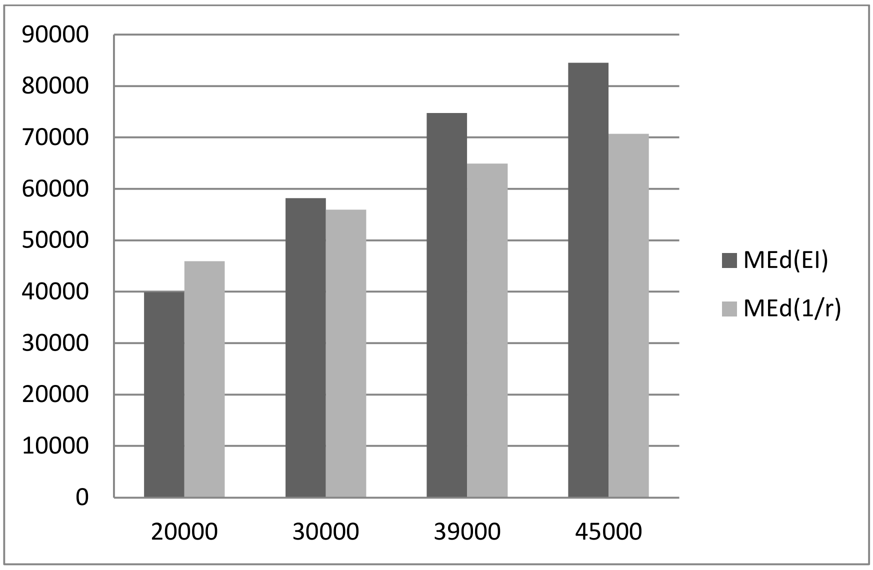


Fig. 3.14 - Simplified method comparison, dependency on bending moment, bridge pier, length 27 m

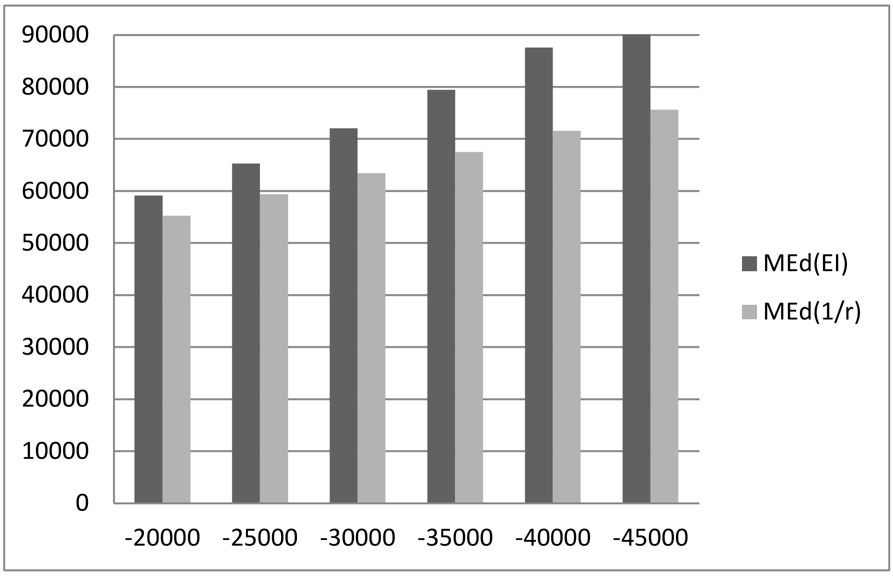


Fig. 3.15 - Simplified method comparison, dependency on axial force, bridge pier, length 27 m

3.4.10. Biaxial bending (5.8.9 [1])

The following provisions apply when simplified methods are used.. according to 5.8.9 (2) the imperfection Imperfections need to be taken into account only in the direction where they will have the most unfavourable effect. It means into direction of the bigger eccentricity of normal force.

Principe of recalculation of normal force position is given in **Remark**.
No further check is necessary if the slenderness ratios satisfy the following two conditions given in [1] art. 5.8.9 (3), expressions (5.38a) and (5.38b)

$\lambda_y/\lambda_z \leq 2$ and $\lambda_z/\lambda_y \leq 2$

and if the relative eccentricities e_y/h a e_z/b (see Fig 5.8 [2]) satisfy one the following conditions:

$\frac{e_y/h_{eq}}{e_z/b_{eq}} \leq 0{,}2$ or $\frac{e_z/b_{eq}}{e_y/h_{eq}} \leq 0{,}2$

Where

b, h are the width and depth of the section;

$b_{eg} = i_y\sqrt{12}$ a$h_{eg} = i_z\sqrt{12}$ for an equivalent rectangular section;

λ_y, λ_z are the slenderness ratios l_0/i with respect to y- and z-axis respectively;

i_y, i_z are the radiuses of gyration with respect to y- and z-axis respectively;

$e_z = M_{Edy} / N_{Ed}$; eccentricity along z-axis;

$e_y = M_{Edz} / N_{Ed}$; eccentricity along y-axis;

M_{Edy} is the design moment about y-axis, including second order moment;

M_{Edz} is the design moment about z-axis, including second order moment;

N_{Ed} is the design value of axial load in the respective load combination.

If the condition of Expression (5.38) is not fulfilled, biaxial bending should be taken into account including the 2nd order effects in each direction.

From stated above can be concluded, that if the section is symmetrical and loads are almost same in both directions, we must check bending at all directions.

Remark: Recalculation of normal force location

Recalculation in z-direction (for bending moment M_y). Analogically, there are defined expressions for z-direction.

Basic eccentricity including imperfection effects:
If the following expression is valid
$\lambda_y > \lambda_z$,
then
$e_{oz,1} = e_{iz} + e_z \geq max(h/30, 20mm)$,
in other case minimum eccentricity is taken account
$e_{oy,1} = e_y \geq max(h/30, 20mm)$

$e_{oz,2} = e_z + max(e_{iz,}e_{iy}) \,.\, e_z/ (e_z^2 + e_y^2)^{0,5} \geq max(h/30, 20mm) \,.\, e_z/ (e_z^2 + e_y^2)^{0,5}$

Resulted basic eccentricity:
$e_{oz} = max(e_{oz,1}, e_{oz,2})$

Where

e_y, e_z basic eccentricity of normal force

e_{iz}, ei_y eccentricity from imperfections

Second order effects eccentricity:
If the expression
$\lambda_y > \lambda_z$,
Then
$e_{2z,1} = e_{2z}$,
In other case it is not taken account
$e_{2z,1} = 0$

$e_{2z,2} = max(e_{2z}, e_{2y}) \, . \, e_z / (e_z^2 + e_y^2)^{0,5}$

Total second order eccentricity:
$e_{2z} = max(e_{2z,1}, e_{2z,2})$

Where
e_y, e_z basic normal force eccentricity
e_{2z}, e_{2y} second order eccentricity

4. Literature

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