Moment of Resistance of Prestressed Section
Comparison of Hand Calculations and IDEA StatiCa Prestressing

Content

Hand calculation 2
IDEA StatiCa calculation 4
Literature 4

Hand calculation

Moment of resistance is calculated on the example of an “I” section subjected to the combination of tensile axial force and bending moment due to the external load and prestressing (including secondary effects), see the copy of pages from in [1] below.

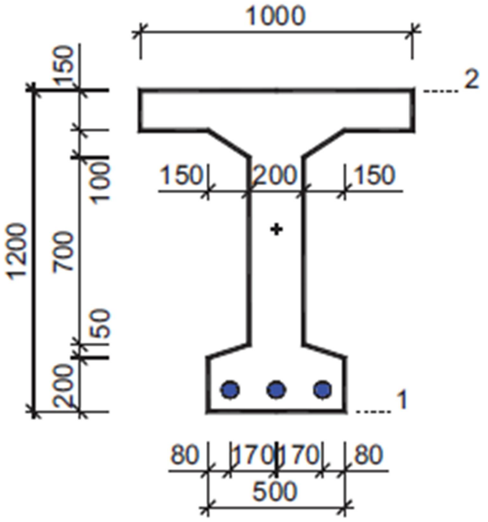


Fig. 8-6 Cross-section dimensions

Now we will document the presented theory on the example of an “I” section, see Fig. 8-6, subjected to the combination of tensile axial force and bending moment due to the external load and prestressing (including secondary effects). The cross-section made of concrete grade C40/50 is prestressed by three tendons, each of which consists of seven strands Y1860S7-12.5. Bilinear stress-strain without hardening is assumed for prestressing steel. The diameter of the tendon ducts is 60 mm, the eccentricity of the tendons related to the centroid of concrete cross-section is $e_p = z_p =$ 0.6022 m. The basic data needed for the calculation of the flexural resistance are in Table 8-4 (input data in red). Let us assume the designation of fibres 1 for bottom fibres, and 2 for top fibres. Then z_1 is the distance of centroid and bottom fibres, and z_2 is distance of centroid and top fibres. The objective is to find ultimate limit strength of the cross-section in accordance with EN 1992-1-1. Following partial factors for materials will be considered: $\gamma_c = 1.5$, $\gamma_s = 1.15$, coefficient $\alpha_{cc} = 1.0$. Note that E_{cd} in Table 8-4 is elastic modulus of concrete calculated from ULS stress-strain relationship from the slope of the line connecting the origin of the stress-strain diagram and the point representing the strain at reaching the maximum strength of concrete, see also chapter 2.1.5. Prestressing secondary effects, permanent load effects, and variable load effects listed in Table 8-4 are all related to the centroid of concrete cross-section.

Table 8-4 Input data for post-tensioned I cross-section

Cross-section characteristics					
Concrete		Concrete - net		Transformed	
A_c [m^2]	0.4425	$A_{c,net}$ [m^2]	0.43402	E_p/E_c	5.53655
I_c [m^4]	0.078025	$I_{c,net}$ [m^4]	0.07489	A_i [m^2]	0.44483
z_1 [m]	0.6822	$z_{1,net}$ [m]	0.69397	I_i [m^4]	0.07562
z_2 [m]	0.5178	$z_{2,net}$ [m]	0.50603	W_{1i} [m^3]	0.10897
W_1 [m^3]	0.11437	$W_{1,net}$ [m^3]	0.10791		
W_2 [m^3]	0.15069	$W_{2,net}$ [m^3]	0.14799		

Material properties				Actions	
Concrete		Prestressing steel		$\sigma_{p,28}$ [MPa]	1080.0
E_c [MPa]	35220.5	E_p [MPa]	195000	$N_{pp,28}$ [kN]	-2109.2
f_{cd} [MPa]	26.667	f_{pk} [MPa]	1860	$M_{pp,28}$ [kNm]	-1270.2
ε_{cu3}	-0.0035	$f_{p0.1k}$ [MPa]	1640	N_{ps} [kN]	100.0
ε_{c3}	-0.00175	f_{pk}/γ_s [MPa]	1617.4	M_{ps} [kNm]	200.0
f_{ck} [MPa]	40.0	f_{pd} [MPa]	1426.1	N_g [kN]	-1000.0
f_{cm} [MPa]	48.0	A_p [m2]	0.001953	$M_{y,g}$ [kNm]	1000.0
E_{cd} [MPa]	15238.1	e_p [m]	0.6022	N_q [kN]	2000.0
				$M_{y,q}$ [kNm]	1296.5

Prestressed Concrete Structures, Jaroslav Navrátil, page 150

Table 8-5 Calculation of flexural resistance of the cross-section

Total permanent forces		Total external load effects	
N_{p+g} [kN]	-3009.2	N_{Ed} [kN]	1000.0
$M_{y,p+g}$ [kNm]	-70.2	M_{Ed} [kNm]	2296.5
Decompression state		**Resistance (primary effects)**	
$\varepsilon_{p,28}$	0.0055385	F^0_p [kN]	2304.16
σ_{cp} [MPa]	-7.799	$F^0_p e_p$ [kNm]	1387.57
σ_p^0 [MPa]	1179.8	N^0 [kN]	3304.16
ε_p^0	0.0060503	M^0 [kNm]	387.57
Calculation of ultimate limit state			
f_{cd} [MPa]	-26.667	N_c [kN]	-1685.1
ε_p	0.05217608	z_c [m]	0.48620
σ_{cc} [MPa]	-26.667	z_p [m]	0.6022
$\Delta\varepsilon_p$	0.046126	$\Delta\sigma_p$ [MPa]	246.3
tg φ_y	0.044309	ΔN_p [kN]	481.0
x [m]	0.078991	M_R [kNm]	1108.98
Reliability condition			
$M_{Ed} + M_{ps}$ [kNm]	≤	$F^0_p e_p + M_R$	eq. (8.21)
2496.5	≤	2496.5	**OK**

The calculation of the ultimate limit strength of the cross-section is carried out in the Table 8-5 using the assumptions and the theory explained in Fig. 8-4 and in formulas above. We assume that $\varepsilon_{cc}=\varepsilon_{cu3}$. Key point is to find total strain in prestressing reinforcement ε_p. It has to be chosen so that the condition of the equation (8.14) is met. Then the flexural resistance M_R is calculated using the equation (8.15). Considering the bending moment $F^0{}_p e_p$ be a part of the resistance of the cross-section, the reliability condition (8.21) has to be fulfilled. Secondary effects of prestressing have to be included into total load effects therefore they are added to the sum of bending moments due to permanent and variable loads. As it follows from the last row of Table 8-5 the combination of loads cause exactly the ultimate limit state of the cross-section.

8.2.3 Universality of solution

The previous chapters solved the problem of the analysis of the action of variable loads on the cross-section subjected to the initial stress due to the permanent load, prestressing, creep and shrinkage of concrete in an elegant way through the introduction of the state of decompression, which the calculation of the ultimate resistance of the cross-section was based on. Up to now, we have not tackled the issue of universality of such a solution.

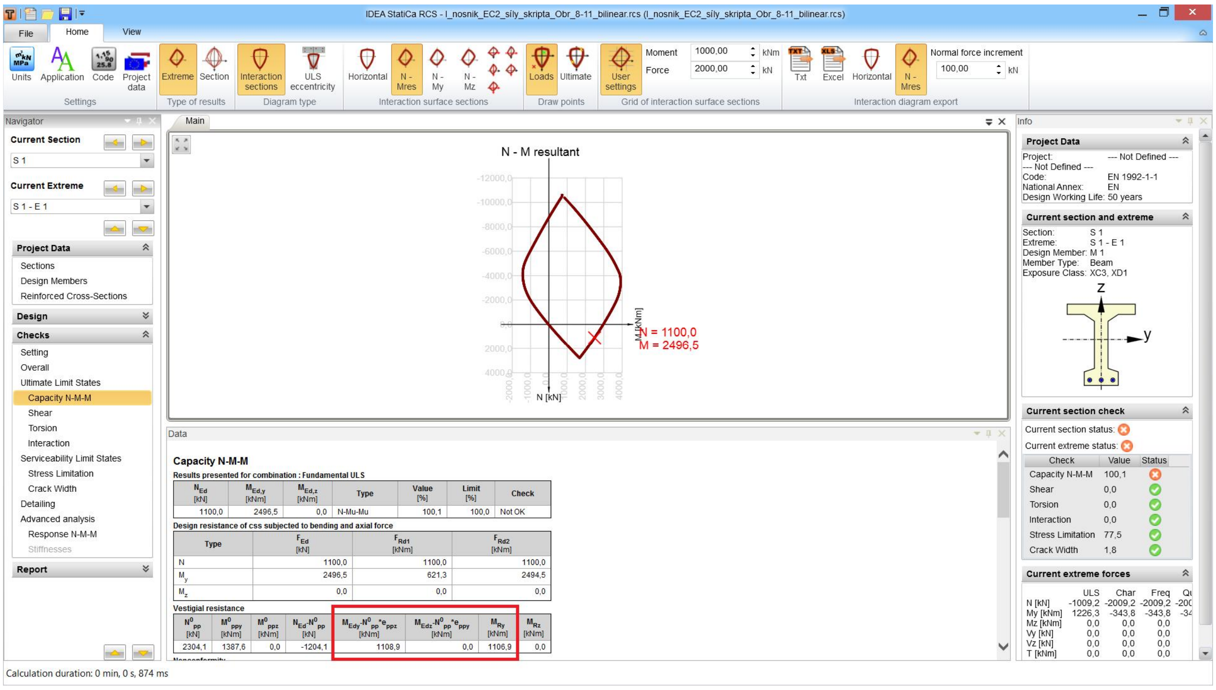
It was explained that the reliability condition is based on the state of decomposition in which the cross-section is subjected to external load forces and these are compared to the internal forces in the cross-section (resistance). In chapter 8.1 this state was introduce by imaginary tensile force N^0 producing zero stress in the concrete. Let us deal with a more general ***example of a statically indeterminate structure*** as in Fig. 8-7.

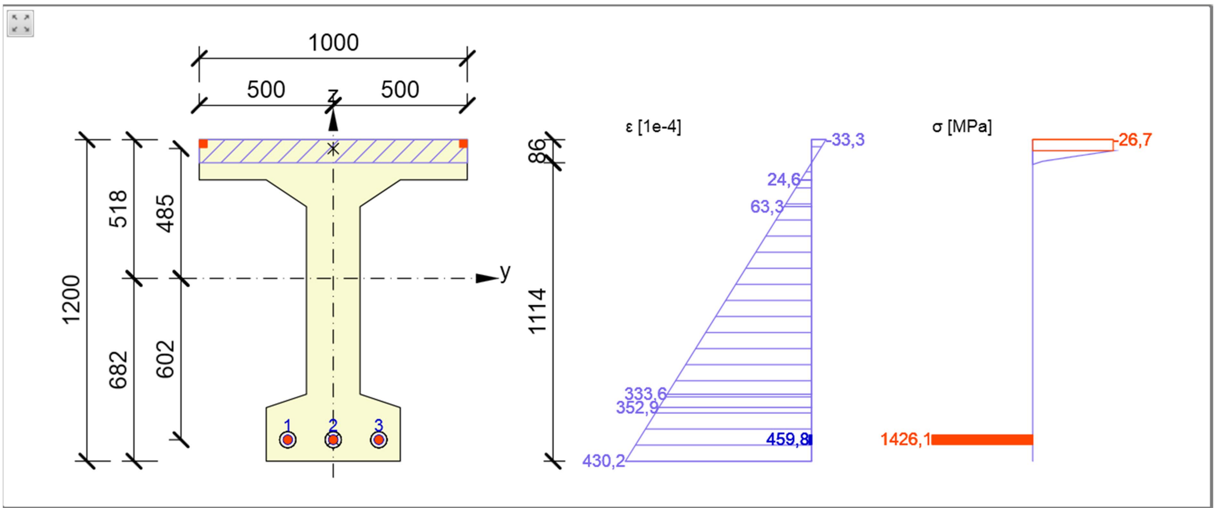
Let us assume a post-tensioned prestressed two-span continuous beam cast on a fixed falsework. After stressing of the tendons ***prior to*** their ***anchoring,*** all the loads, i.e. self-weight g_0 and the equivalent load due to prestressing, ***act on the net concrete cross-section weakened by the tendon ducts*** and produce the stress-state in the cross-sections as schematically shown in Fig. (a). On anchoring of the prestressing reinforcement, the tendon becomes a part of the beam and resists any subsequent loads together with the concrete.

Prestressed Concrete Structures, Jaroslav Navrátil, page 151

IDEA StatiCa calculation

Below you can see the results of IDEA StatiCa calculation. In spite of the fact that the methods used for the calculation are completely different, the results are almost identical.





Literature

[1] NAVRÁTIL, J. *Prestressed Concrete Structures*. VŠB – Technical university of Ostrava, Faculty of Civil Engineering. 2014. Second edition. ISBN 978-80-248-3625-6. 220 pp.