Concrete material model designated for 3D version of IDEA StatiCa® DETAIL

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Introduction

The presented material model is a multi-surface plasticity model designed for monotonic loading. The model calculates the material response in terms of stress for a given strain at a material point. This response is determined by plasticity surfaces and a material law (stress-strain curve). Surfaces restricting compressive behavior correspond to the Mohr-Coulomb model, while the tensile part is restricted by planes corresponding to the Rankine model. The material model is designated for use in applications that calculate the response of reinforced concrete structural details. Therefore, tensile load-bearing capacity is neglected.

Material parameters

The material model parameters are the following:

f_c $\cdots$ compressive strength (input)
f_t $\cdots$ tensile strength (≈ 0)
ε_2 $\cdots$ compressive strain at the end of parabolic curve (default $2 \cdot 10^{-3}$)
E $\cdots$ elastic modulus $E = 2\, f_c/\varepsilon_2$ according to EC 1992-1-1
ν $\cdots$ Poisson's ratio (default $= 0.19$)
φ $\cdots$ angle of internal friction (set to zero as a conservative assumption)

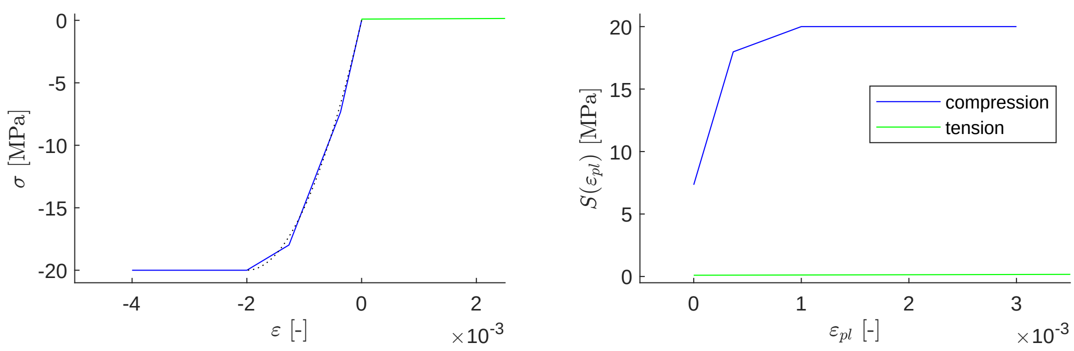

Figure 1: Example of uni-axial material properties: Left: Stress-strain relation, Right: Plastic strain evolution.

The model's constitutive relation is shown in Fig. 1. The compressive part of the constitutive law is a parabolic curve. The tensile strength can be, in general, nonzero, however, since the model is designated for calculation of the reinforced concrete, the tensile strength of concrete is neglected. It is therefore not suitable for calculation of e.g. plain concrete response.

Plasticity model

To calculate the stress in a material point, the stress state must satisfy the following conditions

$$f = \sigma_{eq} - S(\varepsilon_{pl}) \leq 0 \qquad \varepsilon_{pl} \geq 0 \tag{1}$$

where, σ_{eq} represents the equivalent stress, and $S(\varepsilon_{pl})$ is the stress based on material plasticity curve (Fig. 1 right). The following stress equilibrium condition must hold

$$\boldsymbol{R} = \boldsymbol{D_e}\,\boldsymbol{\varepsilon} - \boldsymbol{\sigma} - \boldsymbol{D_e}\left(\frac{\partial f}{\partial \sigma}\varepsilon_{pl}\right) = \boldsymbol{0} \tag{2}$$

First, we check whether the yield criteria are met. If not, the response is elastic. In the opposite case (the elastic limit is exceeded), material response is nonlinear and nonzero plastic strain ε_{pl} is obtained.

To illustrate it using a simple 1D case, stress and strain tensors result in scalars, and the elastic stiffness represented by $\boldsymbol{D_e}$ correspond to the elasticity modulus E. This scenario is depicted schematically in Fig. 2. Our goal is to determine the plastic strain value ε_{pl} that fulfills the equilibrium - Eq. (2).

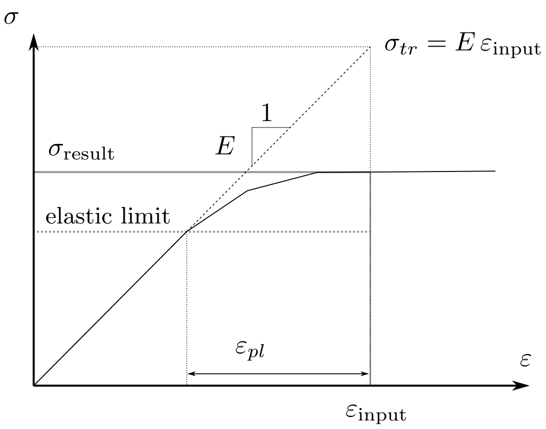

Figure 2: Uni-axial case stress equilibrium: $R = E\,\varepsilon - \sigma - E\,\varepsilon_{pl} = 0$

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Multi-surface plasticity model

In a general multi-surface plasticity model, each surface can have its own yield function. In our case, there are six surfaces for concrete in compression and three surfaces that restrict the tensile behavior. However, as the solution is performed in the principal stress space, where values are arranged from highest to lowest, the solution space is significantly reduced. The approach to calculating equivalent stress differs for tensile and compressive surfaces.

$$\text{tension:} \qquad \sigma_{et} = \sigma_{\max} \tag{3}$$

$$\text{compression:} \qquad \sigma_{ec} = (1+\beta)\,\sigma_{\max} - \sigma_{\min} \qquad \beta = \frac{2\,\sin(\varphi)}{1-\sin(\varphi)} \tag{4}$$

Since the constitutive law differs in compression and tension, also Eq. (1) needs to be split into two cases

$$f_t = \sigma_{et} - S_t(\varepsilon_{plt}) \leq 0 \qquad \varepsilon_{plt} \geq 0 \tag{5}$$

$$f_c = \sigma_{ec} - S_c(\varepsilon_{plc}) \leq 0 \qquad \varepsilon_{plc} \geq 0 \tag{6}$$

Because both ε_{plt} and ε_{plc} can be nonzero, Eq. (2) needs to be enhanced as well

$$\boldsymbol{R} = \boldsymbol{D_e}\,\boldsymbol{\varepsilon} - \boldsymbol{\sigma} - \boldsymbol{D_e}\left(\frac{\partial f_t}{\partial \sigma}\varepsilon_{plt} + \frac{\partial f_c}{\partial \sigma}\varepsilon_{plc}\right) = \boldsymbol{0} \tag{7}$$

Elastic envelope

Let's take a look at Fig. 3, which illustrates the elastic envelope in the space of principal strains and stresses. For a positive, non-zero value of the angle of internal friction, $\varphi > 0$, the pyramid defined by the Mohr-Coulomb plasticity surfaces widens as compressive strains or stresses become more negative. The slope of the Mohr-Coulomb plasticity surfaces for compression is determined by the angle of internal friction, φ. If the angle is zero, the shape of the compressive part of the elastic envelope is prismatic. The normals of the tensile plasticity surfaces align with the principal stress space bases, with each surface intersecting a specific base at the tensile elastic limit defined by the material law. Once the elastic limit is surpassed, the stress for a given strain is determined iteratively using a Newton process.

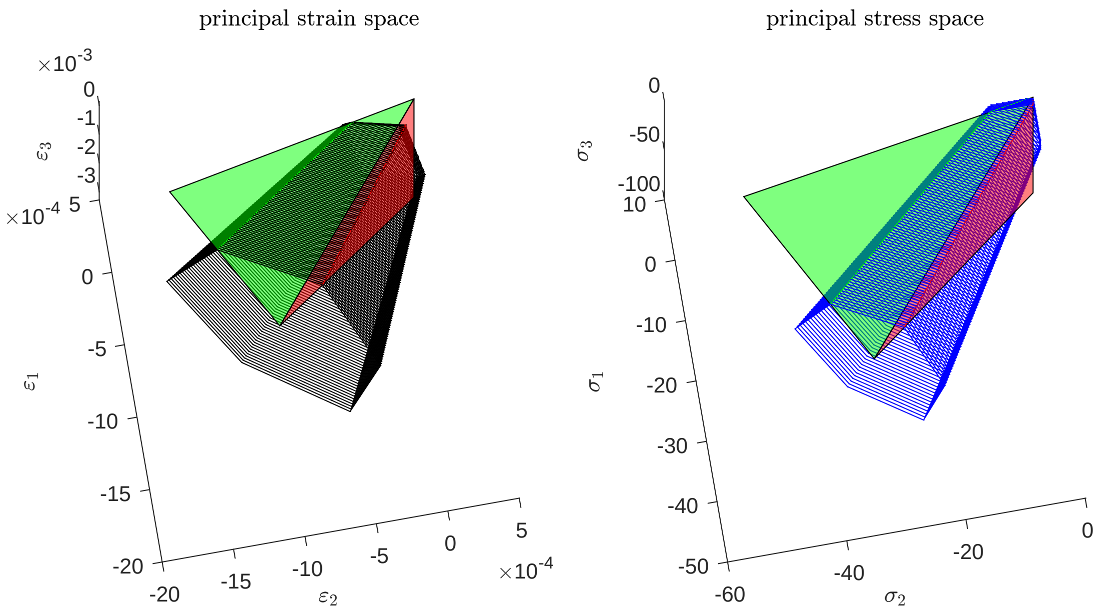

Figure 3: Elastic envelope. Left: in the principal strain space. Right: in the principal stress space.

Remarks on tri-axial compression

In regions subjected to local loading, thanks to the model tri-axial nature, stresses surpassing the material strength are anticipated. Let us denote the effective strength variable $f_{c,\text{eff}} = \kappa\ f_c$. The rise in the effective material strength is clearly depicted in Figure 4. If we seek a value compliant with the material strength, the equivalent stress can be used, or the actual stress need to be compared with the effective strength.

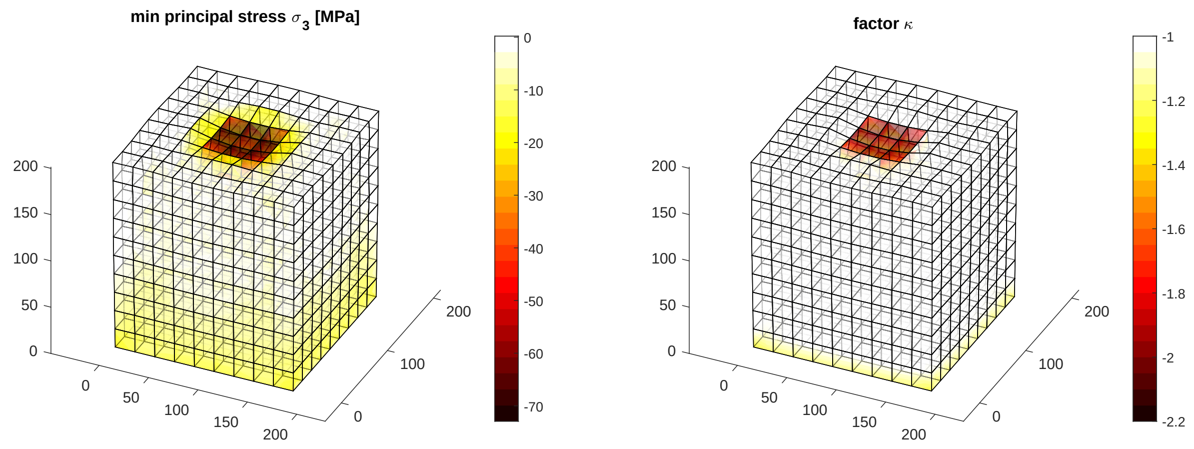

Figure 4: Model results for footing analysis. Left: Minimum principal stress distribution. Right: Factor κ values in locally loaded areas.

The material model's behavior under tri-axial compression in terms of stress and strain is demonstrated in the graphs in Fig. 5. These graphs illustrate the evolution of the minimum stress as the minimum strain decreases for various tri-axial stress ratios. In the case of the ratio $\sigma_1/\sigma_3 = 1$, the elastic limit is never exceeded, allowing stress to grow infinitely. The graphs show that significant change in material behavior under tri-axial compression can be achieved by a change in value of the angle of internal friction. In the Detail application, the angle of internal friction is set to zero by default. This configuration should suffice for structural details and is on the conservative side.

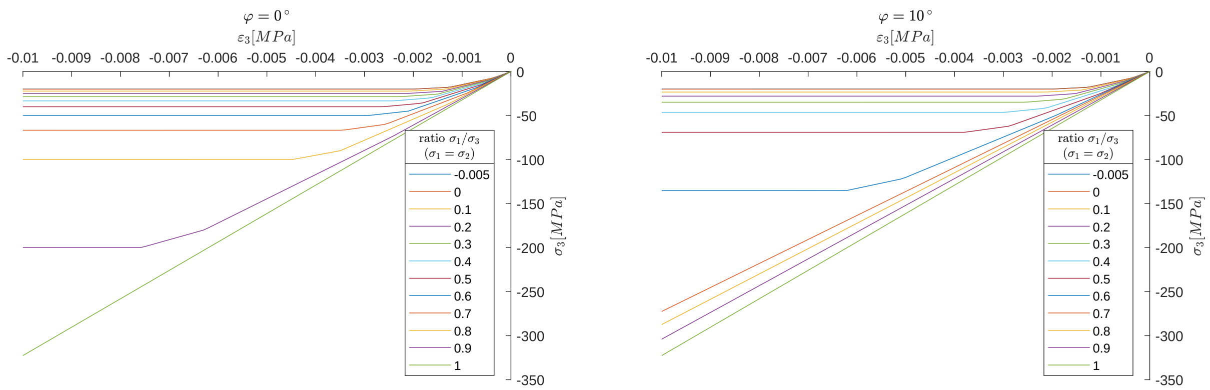

Figure 5: Evolution of minimum stress for various tri-axial stress ratios for models with angles of internal friction $\varphi = 0°$ and $\varphi = 10°$.